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translated from: KoMaL, April 2005 supplement, pages 49-50

Pdf file: KoMaL, April 2005 supplement, pages 49-50

From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

## Part I

1. Find the possible values of the positive integers  $a$  and  $b$ , if  $a < b$ , and whenever  $x$  and  $y$  are elements of the interval  $[a, b]$ , then  $\frac{1}{x} + \frac{1}{y}$  is also an element of  $[a, b]$ .

(12 points)

2. a) Solve the equation

$$\sqrt{1 + x\sqrt{x^2 - 16}} = x - 1$$

over the real numbers.

(5 points)

- b) Find the domain of the above equation.

(7 points)

3. A car dealer sells five kinds of cars. It sells one car of each kind for

2.3, 2.8, 3, 3.1, 3.5

million Ft, and its profit (the difference of the selling price and the purchase price) is, for the kinds in the order above,

3, 2.5, 2.4, 2.1, 2

percent of the selling price. These two lists of data do not change during the investigation. Of each kind (in the order above) it sold

in 2003    25    32    27    38    19 cars,

in 2004    27    31    29    32    33 cars.

- a) What was its profit in 2003, and in 2004?

(5 points)

- b) If in 2005 it sells only one kind of car, which kind (with what unit price) is it worth selling, and at least how many of them must it sell, if it wants to reach the profit of 2004 by selling as few cars as possible?

(7 points)

4. The circles of radii  $r$  and  $R$  are externally tangent to each other, and  $r < R$ . The angle of their common external tangents is  $\alpha$ .
- a) What is  $\frac{R}{r}$ , if  $\alpha = 60^\circ$ ? (5 points)
- b) What is  $\alpha$ , if  $\frac{R}{r} = \frac{5}{3}$ ? (5 points)
- c) What is the area of the quadrilateral two of whose vertices are the centers of the circles, and the other two are the points of tangency on one of the common external tangents, if  $R = 5$  and  $r = 3$ ? (5 points)

## Part II

5. The set  $H$  is a subset of the set of positive integers not greater than 2005 such that the sum of any two of its elements is not divisible by 3. At most how many elements does  $H$  have? (14 points)
6. There are five red balls in a box.
- a) How many white balls must we put into the box, if we want that drawing one ball at random from the box (each ball equally likely), the ball drawn is red with probability 0.25? (6 points)
- b) At least how many white and at least how many black balls must we put into the box (at least one of each), if we want that drawing one ball at random, the ball drawn is not black with probability 0.25? (10 points)
7. Solve the inequality
- $$\log_x (x^2 + x - 4) < 1$$
- over the real numbers. (16 points)
8. Let  $A$  be the right triangular prism whose base is an equilateral triangle with side 2 and whose lateral edges are also 2 long. Rotate the solid  $A$  by  $90^\circ$  around the line through the center of one of its lateral faces, perpendicular to that face, and let  $A'$  be the rotated solid. What is
- a) the volume of the solid  $A \cup A'$ ? (8 points)
- b) the surface area of the solid  $A \cup A'$ ? (8 points)
9. a) Write the equations of the parabolas whose graph is tangent to the line  $y = 2 - 2x$  at the point  $P(1, 0)$ , and whose axis is parallel to the  $y$ -axis. (9 points)
- b) Where are the vertices of these parabolas? (9 points)