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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. Find the four consecutive odd numbers the sum of whose squares is 48 more than the sum of the squares of the even numbers between them.
2. On the sides AB , BC , CD , and DA of the unit square $ABCD$ we take the points E , F , G , H so that $AE = \frac{1}{2}$, $BF = \frac{1}{3}$, $CG = \frac{2}{3}$, and $DH = \frac{1}{2}$. Find the angles, the perimeter, and the area of the quadrilateral $EFGH$.
3. We form six-digit numbers from the digits 1, 2, 3.
 - a) How many different numbers can we get?
 - b) How many of them contain each of the three digits at least once?
 - c) What is the probability that the number we get is even?
 - d) What is the probability that the number we get is divisible by 3?
4. Solve the following equation over the real numbers.

$$\log_{\sin x} (1 - \cos^2 x) = 2^{\sqrt{4-x^2}}$$

Part II

5. Péter's library of 1000 volumes consists of books in Hungarian, English, and German. $p\%$ of the books are in Hungarian, $p\%$ of the foreign-language books are in English, and he has only 10 books in German. Find the number of Hungarian and English books.

6. Let

$$f(x) = \frac{1 - \frac{4x}{x+1}}{\frac{x}{x+1} - \frac{1}{1-x} - \frac{2x}{x^2-1}}$$

- a) What is the domain of the expression above?
 - b) Bring the expression to the simplest possible form.
 - c) Through how many lattice points does the graph of the function $f(x)$ pass?
7. A statue consists of two balls placed on top of each other, where the radius of the upper ball is half the radius of the lower ball, and the height of the statue is 3 meters. To protect the statue against the hardships of winter, a competition was announced for making a "protective cover". Two entries arrived: one proposed a cover in the shape of a frustum of a square pyramid, the other in the shape of a frustum of a cone. In both, the lateral surface touches both balls, and the top touches the smaller ball (there is no bottom base). The cost of the cone frustum design is 12 000 Ft/m², and that of the pyramid frustum is 10 000 Ft/m². Which design would be cheaper to make?

8. Two trucks arrive at a logistics center at random times within a 24-hour period. Unloading starts as soon as a truck arrives; it takes 1 hour for one of them and 2 hours for the other. If the second truck arrives while the other is still being unloaded, it has to wait until the unloading is finished. What is the probability that one of the trucks has to wait for unloading?
9. The n th term of a sequence is $a_n = \frac{2}{(2n+1)(2n+3)}$, where n is a positive integer.
- a) Find b and c so that $a_n = \frac{b}{2n+1} - \frac{c}{2n+3}$ for every n .
- b) Find the fifth decimal digit of the sum of the first 2005 terms.