

Standard Level Practice Set I - 2005 (April 2005 supplement)

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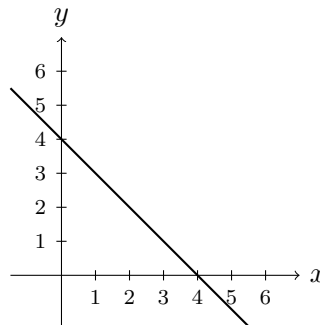
From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the standard-level exam takes 180 minutes. Part I (45 minutes, 30 points) consists of short problems; only the final answer has to be given, except where the problem asks for a justification. Part II (135 minutes) consists of Part II/A, three problems worth 36 points in all, and Part II/B, three problems of 17 points each, of which students choose and solve 2, for 34 points. So there is a total of 100 points possible. Calculators and four-digit function tables may be used.

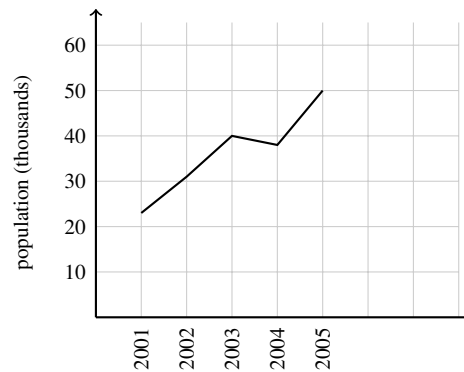
This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present the problems.

Part I

1. In the store we can buy 4 kilograms of bread for 500 forints. How many kilograms of bread can we buy for the same money, if the price of bread is raised by 20%? *(2 points)*
2. Find the largest subset of the real numbers on which the expression $\frac{1}{\sqrt{x^2 - 4}}$ is defined. *(2 points)*
3. We put cards with the numbers 1, 2, 3, 4, 5, 6 into an urn, and draw two of them without replacement. Writing the drawn numbers next to each other in the order we choose, we form a two-digit number. What is the probability that we get a number divisible by 6 this way? *(3 points)*
4. The points $A(3, -6)$ and $B(7, -3)$ are given in the coordinate system. Find the length of the segment AB . *(2 points)*
5. Give the rule of the linear function shown in the figure. *(2 points)*



6. The following seven numbers are given: 3, 2, 3, 7, 1, 9, 10. Find the mean and the median of the data. *(2 points)*
7. Smallcountry has seven settlements in all. The capital is connected to every settlement by a road, and apart from the capital, from every settlement one can get directly to two other settlements by road. Draw a simple graph that represents the road network of Smallcountry. *(2 points)*
8. The graph shows the population of a town in the years indicated. Determine, for the period shown,



- a) between which two years the increase in population was the largest, and approximately how large this increase was; (2 points)
- b) between which two years the population decreased. (1 points)
9. “Every spotted magpie has a spotted tail.” Which of the following statements means the same as this statement? (3 points)
- A) There is a spotted magpie whose tail is spotted.
- B) Every magpie with a spotted tail is spotted.
- C) There is no spotted magpie whose tail is not spotted.
- D) Every magpie has a spotted tail.
- E) For every magpie it is true that if its tail is spotted, then it is spotted itself.
- (The original Hungarian sentence, “Minden tarka szarka farka tarka”, is a well-known tongue twister.)
10. a) Simplify the following algebraic fraction. (2 points)
- $$\frac{a^2 - b^2}{a + b} \cdot \frac{a + b}{5a - 5b}$$
- b) Find the value of the expression for $a = 150$, $b = 55$. (1 points)
11. One leg of a right triangle is 5 cm, and the other leg is 12 cm long.
- a) What is the area of the triangle? (1 points)
- b) What is the altitude to the hypotenuse? Justify your answer. (2 points)
12. Graph the function $x \mapsto x^2 + 2x - 3$ on the interval $[-2, 2]$. (3 points)

Part II/A

13. A company rents out photocopiers. There are two types: for the Mismásoló the monthly rent is 5000 Ft and 8 Ft must be paid per copy; for the Másmásoló the monthly rent is 9500 Ft and 5 Ft must be paid per copy. (There are no other costs.)
- a) By what percent is copying more or less expensive with the Mismásoló than with the Másmásoló, for 1000 copies a month? (2 points)
- b) As a function of the monthly number of copies, which copier is it worth choosing? (4 points)
- c) We want to open a copy shop, and we expect to make 20 000 copies a month there. What price should we set for one copy, if we want a 20% profit compared to our copying costs? (We rent the Másmásoló for the shop.) (3 points)
- d) In our copy shop we signed a flat-rate rental contract for the machine: we prepay 20 000 copies a month according to the rental terms of the Másmásoló, and therefore we copy as many as we want for this amount. We decide to sell a

copy for 10 forints. In one month we have already sold 10 000 copies, and the last day of the month has come. We receive an order for 5000 copies. For how much should we take one copy, so that we make a 20% profit on the money invested in this month? *(3 points)*

14. a) Graph the function $x \mapsto \log_2 x$ defined on the positive real numbers. *(4 points)*

b) Solve the equation

$$4^{x+\frac{3}{2}} - 33 \cdot 2^x + 2^2 = 0$$

over the real numbers. *(8 points)*

15. The parallel sides of an isosceles trapezoid are 8 and 48 units long. Its legs are equal to the sum of the shorter parallel side and the height.

a) Find the area of the trapezoid. *(6 points)*

b) Find the radius of the circle circumscribed around the trapezoid. *(6 points)*

Part II/B

Choose and solve two of the problems 16, 17, and 18.

16. Solve the following equations over the real numbers.

a) $\sin 2x = \frac{1}{2}$ *(6 points)*

b) $\log_2(x+1) + \frac{1}{2} \log_2(x^2 - 8x + 16) = 1 + \log_2 3$ *(11 points)*

17. After a tanker accident, three ships want to localize an oil slick that got into the sea, with the help of a rope covered with sponge stretched between them. The pairwise distances between the ships are 40, 60, and 80 meters. The oil slick fills the part between the sponge ropes.

a) What is the area of the oil slick? *(7 points)*

b) Of the 20 crew members of one of the ships, 15 refuse to work, citing bad food. The captain randomly assigns 2 sailors to the pump. What is the probability that exactly one of those assigned is a striker? *(4 points)*

c) Of the 80 crew members of the largest ship, 10 are Swedish, 20 Danish, 16 Chinese, and 34 German. Show the composition of the crew in a pie chart. *(6 points)*

18. Two dogs are tied in the coordinate plane: one to the point $(4, 5)$ with a leash of length 2, one end of which can rotate freely around the point; the other to the line $3x + 4y = 7$ with a leash of length 1, one end of which can slide freely along the line. What is the smallest possible distance between the two dogs? *(17 points)*