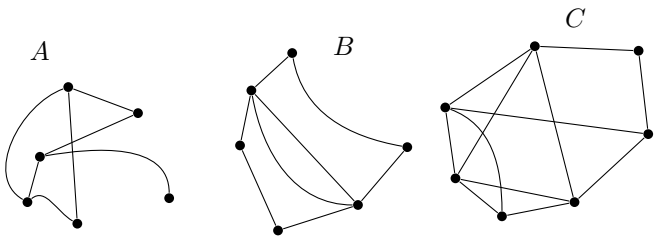
Advanced Level Practice Set - 2005 (December 2005)

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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.
The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.
This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. a) A trail that contains every edge of a graph exactly once is called an Euler trail, and an Euler trail whose starting and ending points coincide is called an Euler circuit. Which of the three graphs in the figure have an Euler trail, and which have an Euler circuit?

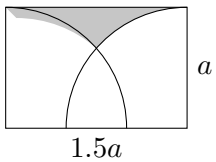


b) What digits can the letters stand for, if $\mathrm{R} = 0$, different letters stand for different digits, and the addition is of course correct?

$$\begin{array}{cccccc} & \mathrm{O} & \mathrm{R} & \mathrm{O} & \mathrm{L} \\ + & & \mathrm{A} & \mathrm{M} & \mathrm{O} \\ \hline & \mathrm{A} & \mathrm{R} & \mathrm{R} & \mathrm{A} \end{array}$$

2. Two vertices of a right triangle are $A\,(7,1)$ and $B\,(7,7)$. Where can the third vertex of the triangle be, if its circumscribed circle passes through the origin? Write the equation of this circle.

3. The sides of the rectangle in the figure are a and $1.5a$. What is the area of the region (shaded dark in the figure) left uncovered by the quarter discs of radius a?



4. The distance of the inaccessible points A and B of a flat terrain has to be determined. So we choose a point C from which the distance AB is seen at an angle of 60°. Walking 50 m along the bisector of this 60° angle, away from the segment AB, we get to a point D. From here, the direction to A makes an angle of 20°, and the direction to B an angle of 10°, with the path we walked. How long is the distance AB?

Part II

5. a) List in increasing order the (positive integer) numbers k for which

$$7 \mid k^3 + 9k^2 + 23k + 15$$

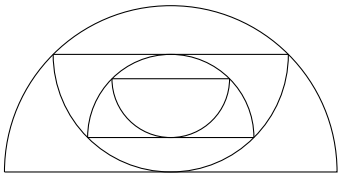
What is the sum of the first 100 terms of the sequence obtained this way?

b) Let $a > 1$. Find the value of the following expression:

$$a^{\frac{\lg a}{\frac{\lg(\lg a)}{\lg a}}}$$

6. A box contains 4 red and 2 yellow balls. We draw without replacement until we draw the first yellow ball. The outcome of the experiment is the number of draws needed for this. Show the distribution of the random variable defined this way, and compute its expected value.

7. We draw new semicircles into a semicircle of diameter 1 dm, as shown in the figure. What are the area and the perimeter of the second semicircle? What kind of sequences do the perimeters and the areas of the semicircles form? Do we get a finite value if we add up the areas of the infinitely many semicircles obtained this way? If yes, what is this sum?



(Each new semicircle is inscribed in the previous one: its diameter is a chord of the previous semicircle parallel to the previous diameter, and its arc touches the previous diameter.)

8. Consider the following function defined on the real numbers: $x \mapsto x^3 + 4x^2 + 5x$.

a) Find the zeros of the function.

b) Does the function have a local extremum? If yes, find it.

c) Describe where the function is increasing and where it is decreasing.

d) Does the function have a point of inflection? If yes, find it.

e) Write the equation of the tangent line to the graph of the function at the point with x-coordinate 0.

9. The table contains data of eight countries.

Country	Area (km^2)	Population	Capital	Population of the capital
Australia	7 682 300	17 483 000	Canberra	197 000
Bangladesh	147 570	111 400 000	Dhaka	3 683 000
Netherlands	33 939	15 298 000	Amsterdam	720 000
India	3 287 263	846 303 000	New Delhi	273 000
China	9 608 378	1 198 340 000	Beijing	7 000 000
Hungary	93 036	10 310 000	Budapest	1 992 000
Malta	316	362 000	Valletta	9210
Mongolia	1 565 500	2 250 000	Ulaanbaatar	619 000

a) List the countries in increasing order of population density.

b) In which country is the probability largest that a citizen lives in the capital?

c) In how many ways can we choose four of the countries listed here so that exactly two European and exactly two Asian countries are among them?

d) Consider the populations of these countries as a set of data. What are the range and the mean of this data set?

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