Advanced Level Practice Set - 2005 (February 2005)

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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.
The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.
This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. a) $A = 101^{100^{99}}$, $B = 99^{100^{101}}$. Which number is larger, A or B?

(7 points)

b) Write the equation of the tangent line to the graph of $f(x) = x^2$ that makes a 60° angle with the positive half of the x-axis.

(6 points)

2. Where is the mistake in the following arguments? What is the solution of the equation in a), and what is the probability in question in b)?

a) The equation $\sqrt{x} = -x$ has no solution, because the square root of a number cannot be negative.

(4 points)

b) What is the probability that, rolling seven fair dice, there is a 1 among the numbers? The probability of rolling a 1 with a single die is $\frac{1}{6}$. So the probability that rolling two dice there is a 1 among the numbers is twice as much, $\frac{1}{3}$; so the probability in question is 7 times as much, $\frac{7}{6}$: the event is more than certain.

(5 points)

c) One meter is 100 centimeters, so $\frac{1}{4}$ meter $= 25$ centimeters. The square root of $\frac{1}{4}$ is $\frac{1}{2}$, the square root of 25 is 5, so $\frac{1}{2}$ meter $= 5$ centimeters.

(4 points)

3. The height, one diagonal, and the other diagonal of a trapezoid are, in this order, three consecutive terms of a geometric sequence with ratio $q = 2$. The area of the trapezoid is $T = \sqrt{60} + \sqrt{12}$ square units. What is the height of the trapezoid?

(12 points)

4. Solve the following inequalities over the real numbers.

a) $2x^4 \leq x^2 + 1$

(5 points)

b) $2\cos^2 x \geq \cos x + 1$

(8 points)

Part II

5. a) The sum of the positive integers A and B is 1000. Prove that the last three digits of A^2 are the same as the last three digits of B^2.

(4 points)

b) Give positive integers A and B with sum 1000 such that the last *four* digits of A^2 are the same as the last *four* digits of B^2.

(3 points)

c) How many positive divisors does the number $H = 2^{24} - 1$ have?

(9 points)

6. A box contains coins and balls, made of silver or gold. 20% of the objects in the box are balls, 40% of the coins are silver, and 80% of the gold objects are coins.

a) What percent of the objects in the box are gold coins?

(4 points)

b) We draw an object from the box at random. What is the probability that it is made of silver?

(4 points)

c) We draw a ball from the box at random. What is the probability that it is made of silver?

(4 points)

d) Interpret the results of b) and c) for the following events: E: the object drawn is made of silver; G: the object drawn is a ball.

(4 points)

7. The first term of an arithmetic sequence is 9, and its ninth term is 33.

a) Find the sum of those terms of the sequence that are between 149 and 301.

(10 points)

b) What is the sum of the squares of the first twenty-five terms of the sequence?

(6 points)

8. In a clinic, a blood test is done to detect a disease that, on average, one person in a hundred has. The people to be tested arrive in groups of fifty, and the practice so far has been to test each of them individually. Testing one blood sample costs 300 forints.

The head of the clinic received two proposals for changing this:

A) Set aside part of each blood sample of a group of 50 in an identifiable way, mix the other parts, and test the group sample first. If it is negative, then obviously everyone is healthy; if positive, test the individual samples set aside.

B) Similarly to A), first test the combined sample of the group of 50. If it is positive, then make two group samples of 25 people each from the individual samples set aside, and test these. If either group turns out to be positive, then test the members of that group individually.

As the head of the clinic, would you change the practice? If yes, how and why?

(16 points)

9. Calculators may *not* be used for the following problems. Justify your results.

a) 1. Write the sum of the first four terms of the expansion of $(1 + x)^{10}$, in increasing powers of x.

(2 points)

2. Using this expression, give an approximation of 1.005^{10}.

(1 points)

3. Prove that the approximation obtained this way is accurate to four decimal places.

(5 points)

b) 1. Sketch the graph of the function $f(x) = \dfrac{1500x + 1}{500x - 2}$ for $x > 1000$.

(2 points)

2. Find the integer part of the positive solution of the equation

$$\frac{1500x + 1}{500x - 2} = \frac{3x}{10\,000}$$

(6 points)

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