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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. A cyclist covered $\frac{1}{4}$ of the distance between the towns A and B in 1 hour, and the rest of the way in 4 hours. His speed (in km/h) was an integer on both parts, and the least common multiple of the two speeds is 72. How long is the distance AB ?

(11 points)

2. a) Solve the following system of equations over the pairs of real numbers.

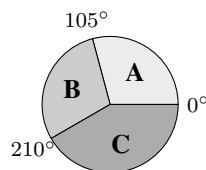
$$\frac{1}{1 + \tan x} + \frac{1}{1 + \cot x} = xy, \quad xy + 4 = 2(x + y)$$

(7 points)

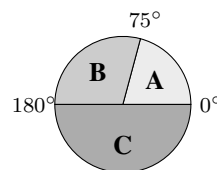
- b) The first term of an arithmetic sequence is the larger of the solutions x, y of the system, and its difference is the smaller one. How many two-digit positive integer terms does the sequence have?

(6 points)

3. Three villages elect a common mayor. There are three candidates (**A**, **B**, and **C**). In village 1, 2016 voters, and in village 2, 1320 voters went to the polls. The pie charts show the results in villages 1 and 2 (the angles mark the boundaries of the sectors). Unfortunately the pie chart showing the result in village 3 was lost.



village 1



village 2

The combined final result of the election: **A**: 1599 votes, **B**: 2169 votes, **C**: 1776 votes.

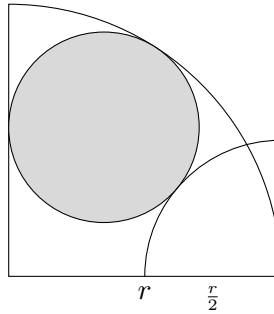
- a) Make the pie chart showing the result of the election in village 3.

(10 points)

- b) How many inhabitants does village 3 have, if 72% of its inhabitants were eligible to vote, and 56% of the eligible voters took part in the election?

(4 points)

4. The figure shows one half of a decorative element of a chapel to be built in a small village (the element consists of two mirror-image copies of this part, on the two sides of a cross). The large quarter circle has radius $r = 80$ cm; a small quarter circle of radius $\frac{r}{2}$ is centered at the other end of its horizontal radius, and the shaded circle (a glass window) touches the vertical radius, the large arc, and the small arc. What is the radius of the shaded circles?



(14 points)

Part II

5. Let A be the domain of the expression (1), and B the domain of the expression (2):

$$(1) \sqrt{2 \sin^2 \pi x - \cos \pi x - 1}, \quad (2) \lg(-3x^2 + 17x - 10)$$

(lg is the base 10 logarithm). Find

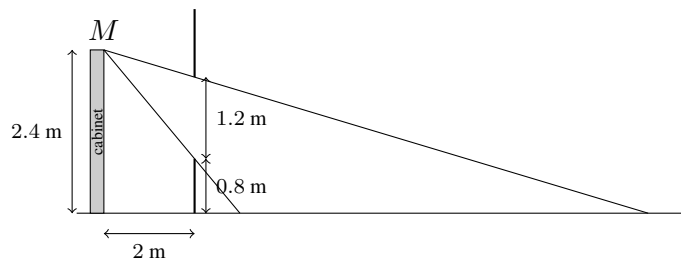
a) the elements of the set $A \cap B$;

(11 points)

b) the elements of the set $B \setminus A$.

(5 points)

6. A bird sits at the corner M of a 2.4 m high cabinet that stands 2 m from a wall, and through a 1.2 m high window in the wall it watches a bug that crawls in the yard, perpendicular to the wall of the house, at a speed of 0.8 m/min. The lower edge of the window is 0.8 m above the ground. For how long does the bird see the bug?



(16 points)

7. For the functions $f(x) = x^2 + ax + b$ and $g(x) = x^2 + bx + a$ ($a > b$) there is exactly one point x_0 at which the tangent lines to the two graphs are perpendicular.

a) Sketch the graph of the function $h(x) = f(x) - g(x)$.

(8 points)

b) Find the area of the region bounded by the graphs of f and g and the y -axis.

(8 points)

8. András saw a chestnut-cream bunny at a very good price in an advertising paper. When he wanted to buy it, he was shocked to find that at the cash register he was charged 66 Ft less than twice the price in the paper. After he complained, it turned out that the first and third digits of the three-digit price had been switched by mistake in the paper. How much does András have to pay, if he buys the item?

(16 points)

9. a) Three parallel lines are given; on each of them we painted 5 points red. Consider all triangles with red vertices that have two vertices on one of the lines and the third vertex on another line; then consider all quadrilaterals with red vertices that have two vertices on each of two of the lines. Which are there more of: triangles or quadrilaterals?

(6 points)

- b) Now let two parallel lines be given, with n red points on one ($n \geq 2$) and $n + 1$ red points on the other. Again we form all triangles and all quadrilaterals with red vertices. Now which are there more of, triangles or quadrilaterals?

(10 points)