

Advanced Level Practice Set - 2005 (March 2005)

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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. In a right triangle the ratio of the legs is 1 : 2, and the perimeter and the area have the same measure. Find the exact length of the altitude to the hypotenuse.

(11 points)

2. Solve the following system of equations.

$$x^2 - y^2 - 6x - 6y = 0, \quad xy - x + y = 0$$

(12 points)

3. In a single round-robin volleyball cup (where any two teams play each other exactly once), after 30 matches had been played, every team still had three matches left. How many teams took part in the cup?

(14 points)

4. The production of a factory grew in five consecutive years: by 12% in the first year, by 15% in the second, and by 20% in the third. In the fourth and fifth years the percentage of growth was the same. In the fifth year the factory reached 2.3 times the production of the year before the period examined. By what percent did production grow in the fourth and fifth years?

(14 points)

Part II

5. a) The roots of the equation $x^2 + bx + c = 0$ are 2 greater than the roots of the equation $x^2 + cx + b = 0$. Find b and c .
b) One root of the equation $x^2 + bx + c = 0$ is 6, and the other root is equal to the discriminant of the equation. Find b and c .
c) The two zeros of the quadratic polynomial $x^2 + bx + c$ are x_1 and x_2 . Write a cubic polynomial whose zeros are $x_1 + x_2$, $x_1^2x_2 + x_1x_2^2$, and $x_1^3 + x_2^3$.

(16 points)

6. Solve the equation $(4 - \cos 8x)(2 + \cos 2x) = 15$.

(16 points)

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7. a) In a class of 27 there are 5 more girls than boys. What is the probability that among three randomly chosen students there are two boys and one girl?
- b) We choose two students from a class of 27. Find the number of boys in the class for which the probability that the chosen students are of different sexes is as large as possible. What is this largest probability?
- c) In class 11 of 27 students, the average of a math test that went badly is exactly 2. The numbers of grades 5, 4, and 3 are equal, and we also know that every grade (from 1 to 5) occurred. What can the mode and the median of the grades be?

(16 points)

8. For a cubic function $f(x)$ we know that $f(1) = f(3) = 4$, the coefficient of x^3 is 1, and $\int_1^5 f(x) dx = 16$. Write the equation of the tangent line to the graph of the function at the point with x -coordinate 4.

(16 points)

9. Three vertices of the convex quadrilateral $ABCD$ are $A(2, 3)$, $B(-2, 5)$, $C(-3, -2)$, and its interior angle at the vertex D is 90° .

- a) Find the area of the triangle ABC .
- b) Find the coordinates of the vertex D so that the area of the quadrilateral $ABCD$ is as large as possible.
- c) What is the perimeter of the quadrilateral $ABCD$, if the point D is on the x -axis?

(16 points)