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translated from: KoMaL, April 2006, pages 208-209

Pdf file: KoMaL, April 2006, pages 208-209

From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.

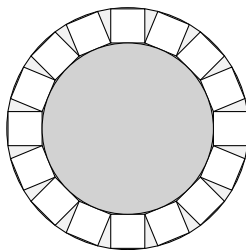
This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. The difference of the sums of the interior angles of two convex polygons is equal to the sum of the interior angles of a heptagon, and one of them has twice as many vertices as the other has sides. How many diagonals do they have together?
2. Solve the following system of equations over the pairs of positive integers:

$$xy^2 - 15y + 56 = 1, \quad y - x = 5$$

3. The roots of the equation $k^2(2 + 4x) - x(x + 1 + k) + 3(x - 2)(2k + 1) - (1 - x)(k + 3) = 0$ are $x_1 = 1$ and $x_2 = 2$. Find the value of k .
4. In a tea house, tea is served so that 16 sugar cubes can be placed around the cup as shown in the figure (the corners of neighboring sugar cubes touch, each touches the cup with one face, and two of its vertices are on the rim of the saucer). How many cm are the edge of the sugar cube and the radius of the cup, if the diameter of the saucer is 10 cm?



Part II

5. How many solutions does the equation

$$|||x| - 1| - 1| - 1| - 1| = \frac{kx}{7}$$

have, depending on the positive integer k ?

6. $ABCD$ is a rhombus with side 10 units. The circle centered at A and passing through the vertex C is touched from inside by the circle centered at B and passing through the vertex D . Find the area of the rhombus.
7. The circle $x^2 + y^2 = 25$, the parabola $y = \frac{1}{3}x^2 + 1$, and the x -axis determine a plane region, which we rotate around the x -axis. What is the volume of the resulting solid of revolution?

8. A company produced solid spherical chewing gum and sold it for 10 forints apiece. Because the raw materials became more expensive, the production cost increased by 300%. The manufacturer would like the ratio of profit to production cost after the sale of the gum not to change, so it raised the price of the gum to 20 forints apiece. The product was also redesigned: the spherical chewing gum of diameter 2 cm now has a concentric spherical cavity inside. Compute the thickness of the wall of the new chewing gum.
9. Solve the following systems of equations:
- a) $\sqrt{x-4} + \sqrt{x-5} - 1 = 7x - x^2 - 2xy$, $\log_5 64^x \cdot \log_4 125^y = 45$;
- b) $xy + y = 2$, $(1-y) \cdot 2 \cos \frac{x\pi}{3} = \frac{x-1}{x+1}$.