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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

## Part I

1. a) A school library of 1600 volumes consists of Hungarian and English books.  $p\%$  of the books are in Hungarian.  $p\%$  of these books can also be found in the library in English translation. The number of books originally written in English is 64. Find the ratio of the Hungarian and English books.  
b) At the end of the year, during the renovation following the purchases, all the books are packed into identical boxes. If we put 25 books in each box, 17 books are left over; if we pack 27 books in each box, then four boxes remain empty, and the last box packed could still hold 5 more books. How many boxes did we have for the packing? By how many books did the library's collection grow by the end of the year?
2. We draw tangent lines to the circle  $x^2 + y^2 = 25$  at its points with  $x$ -coordinate 3.  
a) Write the equations of the tangent lines.  
b) Find the angle between the tangent lines.
3. Which real numbers satisfy the following equation?

$$\log_{4-5x} (4x^2 + 1 - 4x) + \log_{2x-1} (13x - 4 - 10x^2) = 4$$

4. In a family of five, the sum of the ages of the parents is 80 years; the father is the older of them. The ages of the three sons are three consecutive terms of an arithmetic sequence with a prime difference, and their sum is 30. How old are the members of the family, if two years ago the father was three times as old as the oldest son, at whose birth the mother was older than 20?

## Part II

5. A box contains eight balls that are identical to the touch. The number 3 is written on one of them, 2 on two of them, and 1 on five of them. Someone takes out three balls and adds the numbers read from the balls.  
a) What results can be obtained this way?  
b) What is the probability of each result?
6. Two sides of a rectangle are 6 cm and 4 cm long. Where is the point  $P$  on the longer midline from which one of the 4 cm sides is seen at an angle twice as large as the other 4 cm side?
7. The base of a right pyramid with four lateral faces is a square of side  $a$ , and every lateral edge makes a  $30^\circ$  angle with the height of the pyramid. What is the area of the plane figure in which the pyramid is cut by the plane that passes through a vertex of the base of the pyramid and is perpendicular to the opposite lateral edge?

8. The following table shows the distribution of the students' complaints submitted for all subjects about the grading of the advanced-level written exams of the May–June 2006 school-leaving exam.

| Subject                           | Number of complaints | Number of written exams |
|-----------------------------------|----------------------|-------------------------|
| Hungarian language and literature | 1187                 | 6251                    |
| History                           | 1660                 | 11 617                  |
| Mathematics                       | 657                  | 6408                    |
| Biology                           | 855                  | 5530                    |
| Physics                           | 149                  | 1745                    |
| English                           | 145                  | 3956                    |
| Chemistry                         | 218                  | 1814                    |
| Other                             | 614                  | 8700                    |
| Total                             | 5485                 | 46 021                  |

- a) Show the distribution of the complaints in a pie chart.
  - b) What is the range of the number of complaints, and of the number of written exams, for the seven subjects?
  - c) Find the standard deviation of the number of complaints, taking the seven subjects into account.
  - d) In which subject did the students “complain” the most?
  - e) Compute, for the seven subjects, the ratio of the number of complaints (compared to the number of written exams) in percent, and find the mean absolute deviation from the overall ratio.
9. The parabola  $y = ax^2 + bx + c$  passes through the origin, and the slope of its tangent line at its point  $E(3, 6)$  is  $-4$ .
- a) Write the equation of the parabola.
  - b) Give the equation of the tangent line mentioned above.
  - c) In what ratio does the line  $y = 2x$  divide the region bounded by the parabola and the  $x$ -axis?