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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. Solve the following equations over the real numbers:

a) $(4 - x^2) \sqrt{1 - x} = 0$; b) $(0.4^x - 2.5^{x+1} - 1.5) \cdot \log_3(x + 3) = 0$; c) $\sin x = 5 \cos \frac{x}{2}$.

2. Two opposite vertices of the rhombus $ABCD$ are $A(0, 0)$ and $C(2, 4)$. The angles at the vertices A and C are 120° . Find the coordinates of the vertices B and D .

3. Solve the following system of equations over the real numbers:

$$x^3 - x^2y - 4xy^2 + 4y^3 = 0, \quad 2x^2 + y^2 = 12$$

4. The perimeter of the parallelogram $ABCD$ is 26 units, $AB < AD$, the radius of the circle inscribed in the triangle BCD is $\sqrt{3}$ units, and the angle of the parallelogram at the vertex B is 120° . Find the sides of the parallelogram.

Part II

5. Five points are given so that no three of them are on a line. We want to connect them with a network consisting of four straight segments; crossings are allowed. How many such networks are possible?

6. Prove that in every triangle

$$\sin^2 \frac{\alpha - \beta}{2} + \sin \alpha \cdot \sin \beta + \sin^2 \frac{\gamma}{2} = 1$$

where α, β, γ are the angles of the triangle.

7. A flea jumps on a long stick. Each jump is randomly to the left or to the right, and each jump is 10 cm long.

- a) In how many ways can it get, with 10 jumps, 40 cm to the right of the starting point, and 50 cm to the left?
b) Find the probability that after 10 jumps the flea is at each of the points of the stick.

8. In the function $f(x) = (p + 1)x^2 - (p + 3)x + 2p$, p is a real parameter. Find the values of p for which the function takes positive values for every real number x .

9. Find the pairs (x, y) that satisfy the following equations:

a) $\sin^2(x + y) - \cos^2(x - y) = 1$; b) $\frac{1}{\sqrt{2}}(\sin x + \cos x) = 2y^2 - 4y + 3$.