

Advanced Level Practice Set - 2006 (March 2006)

author: József Czinki

translated from: KoMaL, March 2006, pages 140-141

Pdf file: KoMaL, March 2006, pages 140-141

From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. a) Does the following equation have a solution over the positive integers?

$$1! + 2! + 3! + \dots + x! = 2y$$

- b) Solve the following equation:

$$\tan \alpha + \sqrt{3} = \frac{\sqrt{2}}{\cos \alpha}$$

2. The table shows Sanyi's midyear report card (grades from 1 to 5, 5 is the best).

Hungarian literature	5 (excellent)
Hungarian language	4 (good)
History	4 (good)
Mathematics	3 (satisfactory)
English	4 (good)
German	3 (satisfactory)
Physics	2 (pass)
Chemistry	3 (satisfactory)
Biology	2 (pass)
Art	4 (good)
Music	5 (excellent)
Physical education	5 (excellent)

- a) Give the mode, the median, and the range of Sanyi's grades.
b) Compute Sanyi's grade point average (the mean of the grades).
c) Give the standard deviation of the grades.
d) Show the percentage distribution of the grades in a pie chart.
3. Find the positive integers x for which $x^4 + 4$ is a prime.
4. The sum of the first and third terms of a geometric sequence is 40. If we add 4 to the first term and 6 to the second, then together with the third term we get three consecutive terms of an arithmetic sequence (in the same order). Which is this geometric sequence?

Part II

5. In what ratio does the y -axis divide the region bounded by the graphs of the functions $f(x) = x^2 - 4$ and $g(x) = -x^2 - 2x$?
6. The base circle of a horizontal cylinder-shaped tank has radius 1 m, and the length of the cylinder is 3 m. The water in the tank is now 1.6 m deep. We let some of the water out into a vertical cylinder-shaped container that is large enough, with base radius 0.5 m. How high will the water be in this container, if the water level in the horizontal cylinder dropped by 0.3 m?
7. A factory received an order to make 100 cone-shaped ice cream wafers, open at the top, each with a volume of 1 dm^3 . The material is expensive, and the customer only prescribed the volume of the cone. For what vertex angle (the angle of the axial cross section at the apex) of the cone is the material cost minimal?
8. Where can the right-angle vertex of the right triangles be whose area is 25 units and whose hypotenuse has endpoints $A(1, -2)$ and $B(6, 8)$?
9.
 - a) Are there, in a group of 25 people, 3 people who were born in the same month? Justify your answer.
 - b) We choose an arbitrary subset of the set of one-digit positive integers. What is the probability that 1 or 2 is in it?
 - c) At most how many subsets can we choose from the set of one-digit positive integers so that any two of them have a common element?