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translated from: KoMaL, November 2006, page 470

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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. According to Aladár, choosing among the three-digit numbers, and according to Barnabás, choosing among the five-digit numbers, the probability that the chosen number contains the digit 6 is greater. Which of them is right?

(11 points)

2. Two circles with radii 5 cm and 3 cm are given. The centers of the circles are 10 cm apart. How far from the center of the smaller circle is the intersection point of the two common internal tangents of the circles?

(12 points)

3. What is the remainder of $16^{101} + 8^{101} + 4^{101} + 2^{101} + 1$ when divided by $2^{100} + 1$?

(14 points)

4. a) Find the linear functions $f(x)$ for which $2f(x) + 3f(1-x) = 4x - 1$.

- b) What can the sign of c be, if the expression $ax^2 - bx + c$ is positive for every x ?

(14 points)

Part II

5. Give all solutions of the following trigonometric equation in the interval $[0, 2\pi]$:

$$3 \tan \left(x + \frac{\pi}{3} \right) + \tan \left(x - \frac{\pi}{6} \right) = 0$$

(16 points)

6. What is the largest volume of a right circular cylinder that can be inscribed in a right circular cone whose base has radius 5 units and whose height is 7 units? (The cylinder and the cone have a common axis.)

(16 points)

7. A car starts and moves on a straight road with acceleration 2 m/s^2 . A man runs toward the car across the field from a road perpendicular to this road, at a constant speed of 5 m/s. How should he choose his starting direction so that he can jump into the car, if at the moment of starting he is 10 m from the car? After how much time does he reach the car?

(16 points)

8. a) How many simple graphs with five vertices and six edges are there, if we distinguish the vertices, and how many, if we do not?

- b) János drew all graphs with five vertices and six edges, distinguishing the vertices. Lackó happened to pass by and pointed at a vertex. What is the probability that this vertex had degree 1?

(16 points)

9. Solve the following equation over the real numbers:

$$\sqrt[3]{2x^2 - 8x} - \sqrt[3]{2x^2} = 2$$

(16 points)