

Advanced Level Practice Set- 2006 (September 2006)

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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. Solve the following equation: $(x - \sqrt{x-2} - 2)(x - \sqrt{x-3} - 3) = 0$.

(11 points)

2. The function $f : (-1, 5] \rightarrow \mathbb{R}$, $f(x) = \left| (x-2)^2 - 4 \right| - 4$ is given.

a) Give the lattice points of the coordinate plane that lie on the graph of the function.

b) Give the zeros of the function.

c) On which intervals is the function increasing?

(12 points)

3. Two points are given in the coordinate system: $A(1, 5)$ and $B(7, 7)$. Find the coordinates of the point P of the x -axis for which

a) $AP = BP$; b) $AP^2 + BP^2 = 94$; c) $AP + BP$ is minimal.

(14 points)

4. Solve the following system of equations:

$$2^{3x+2} = 4y^3 - 407y + 20, \quad x = \log_2(y - 4)$$

(14 points)

Part II

5. The first term of a geometric sequence is a , and its ratio is q . What is the first term of the geometric sequence whose ratio is q^2 and the sum of whose first 25 terms is equal to the sum of the first 50 terms of the given sequence?

(16 points)

6. A graduating class designed its class picture board in the shape of a centrally symmetric octagon. They wanted to make the octagon from a $1 \text{ m} \times 1.4 \text{ m}$ rectangular board by cutting off four congruent right triangles at the corners of the rectangle. Then the four side lines of the rectangle are also side lines of the octagon. The sides of the octagon obtained this way, in decimeters, are in order: 7, 5, 3, 5, 7, 5, 3, 5. How much would the amount of waste change, if the octagon were cut out from the rectangle determined by the other four side lines of the octagon?

(16 points)

7. Find the smallest positive value x for which $\sin x$ and $\sin 2x$ are the legs, and $\sin 3x$ the hypotenuse, of a right triangle.

(16 points)

8. A department store wants to give a present to those who buy at least seven cans of the liver pâté on sale. In the store the cans are stacked into pyramids with a square base. For example, a pyramid of four layers can be made of $16 + 9 + 4 + 1$ cans. The customers can draw an integer from 1 to 50, with equal probability, by spinning a wheel of fortune. If the number of cans in the pyramid of that many layers is divisible by 7, then the customer gets the present.
- a) The employees of the store built a pyramid of 16 layers. How many cans did they use?
 - b) What is the probability that we spin a number that entitles us to the present?
 - c) One of the customers spun a number such that the number of cans in the pyramid with that many layers was divisible by both 7 and 13. Which number could it have been?

(16 points)

9. What is the volume of the solid of revolution obtained by rotating the graph of the cubic function $f : [-10, 10] \rightarrow \mathbb{R}$, $f(x) = 0.004x(x + 12)(x - 12) + 8$ around the x -axis?

(16 points)