

Advanced Level Practice Set - 2007 (April 2007)

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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. Two cars start at the same time from the towns A and B , which are 24 km apart, driving toward each other. 4 minutes after they meet, the car that started from B arrives in A , and 16 minutes after they meet, the other car arrives in B . Find the speeds of the cars.

(11 points)

2. a) We want to distribute 1000 apples among k students ($k \geq 2$) so that the first student gets x apples, and each further student gets 3 fewer than the previous one, until all 1000 apples are given out with none left over. How many students can there be? How many apples does the last student get?

b) Next time we want to distribute 1000 apples among 16 people so that the first 11 people get the same number of apples, and after that each person always gets 50% more than the previous one. Then 15 apples are left at the end. We record the numbers of apples the people got, which gives 16 data. What are the mean and the mode of these data?

(12 points)

3. Solve the following equations over the real numbers.

a) $\frac{x^2 + 2x + 1}{\sqrt{(x+1)^2}} + x + 1 = 0$

b) $10^{\lg 3x} = 2x - 3$

c) $\sqrt{x-1} \cdot \sqrt{x+5} = \sqrt{x^2 + 4x - 5}$

d) $\sqrt{1 - \sin^2 x} \cdot \tan x = \sin x$

(14 points)

4. One diagonal of a parallelogram is equal to the sine of a certain angle, the other diagonal is equal to the cosine of the same angle, and the shorter side of the parallelogram is equal to the sine of twice that angle. The angle between the two diagonals is 60° . Find the exact values of the sides of the parallelogram.

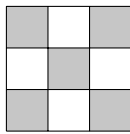
(14 points)

Part II

5. A biology student looks at a watery medium between two microscope slides; the medium covers a square region of side 0.9 mm. A grid of 9 squares, each with side one third of the side of this square, is projected onto the field of view; it is colored gray and white like a chessboard, with the lower left corner gray. Consider two events, when the student first looks into the microscope and sees a point-like bacterium:

A : the bacterium is in a white square;

B : the bacterium is closer to the left side of the square of medium than to its bottom side.



- a) Find the probabilities $P(A)$, $P(B)$, and $P(\overline{A} \cdot B)$.
- b) What is the probability that a spherical dust particle of radius 0.01 mm appears to have a common point with one of the gray squares? (In the microscope, the dust particle looks like a disk of radius 0.01 mm, and its position in the observed medium is random.)
- (16 points)
6. a) Consider the five-digit numbers whose last three digits are different, the sum of these three digits is 5, and none of them is a prime number. What is the sum of those numbers among these that are divisible by 8?
- b) Consider all positive integers n for which $n! + 3$ is a perfect square. How many such perfect squares are there?
- (16 points)
7. The points $A(-2, 0)$ and $B(2, 0)$ are the endpoints of the hypotenuse of a right triangle.
- a) Find the coordinates of the third vertex if the leg opposite the vertex A is 2 units long.
- b) The circle with center B and radius 2 is the inscribed circle of a triangle that has a vertex at A , and the side opposite A is tangent to the circle at the point $E(2 + \sqrt{3}, 1)$. Find the coordinates of the other two vertices of this triangle.
- c) What are the angles of this triangle?

(16 points)

8. Let A , V , and G be the surface area and the volume of a right circular cone, and the surface area of its inscribed sphere. Prove that

$$A^2 G = 36\pi \cdot V^2$$

(16 points)

9. Let $f(x) = x - \frac{2}{3}x^3$, defined on the real numbers.

- a) At which values of x in the interval $[-2, 1]$ does this function take its maximum and its minimum value? What are these values?
- b) On this interval, where is the tangent line of the graph parallel to the graph of the function $g(x) = -3x + 2007$?
- c) What can the rule of the function $h(x)$ be if

$$h(f(x)) = \sqrt[3]{x - \frac{2}{3}x^3}?$$

Find the composite function $f(h(x))$.

(16 points)