

author: József Czinki

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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. A 20 percent (by mass) salt solution (the solvent is water) is heated, and 15 percent of the solvent evaporates. If we now add 0.1 kg of salt to the solution, its concentration (percent by mass) will be the same as if we had added 0.1 kg of solvent and 0.3 kg of salt to the original solution. What was the mass of the original solution?

(11 points)

2. Experience shows that 3% of the button batteries produced in a factory are defective. We select 8 button batteries at random. What is the probability that among them

- a) there is no defective battery?
- b) there are at least two defective batteries?

(12 points)

3. Solve the following equation over the real numbers.

$$\cos^2 x \cdot (2 \cos^2 x + 1) = \frac{105}{128 \cos^4 x + 64 \cos^2 x - 64}$$

(14 points)

4. Sanyi played the Hungarian lottery (5 numbers are marked out of 90, and 5 numbers are drawn). He selected eight numbers (out of the possible 90) and, using only these, played every possible combination of five numbers.

- a) How many X's did he draw on the tickets altogether?
- b) Two of the 8 selected numbers were drawn. How many tickets with two matches did he have?
- c) How many tickets with three matches would he have had if 4 of the 8 selected numbers had been drawn?

(14 points)

Part II

5. Three balls of radius r stand on a horizontal plane, each touching the other two. Find the ratio of the volume of the ball touching all three balls and the plane to the volume of the tetrahedron determined by the four centers.

(16 points)

6. Consider the function $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = |x^3 - 2x^2 - 8x| - 3$.

- a) Where does the function have local extrema?
- b) Find the equation of the tangent line of the function at the point with $x = 3$.

(16 points)

7. Someone deposits 50 000 forints (HUF) in a bank. The first bank would pay 0.4% interest per month for one year, compounded monthly. The second bank would pay 0.3% interest per month for one year, compounded monthly, and in addition, after every fourth month it increases the balance by 0.5%. The third bank pays 3% interest per half year, compounded semiannually.

a) Complete the following table (rounded to one decimal place).

	Balance at end of month 1	Balance at end of month 2	Balance at end of month 3	Balance at end of month 4	Balance at end of month 5	Balance at end of month 6
First bank						
Second bank						
Third bank						
	Balance at end of month 7	Balance at end of month 8	Balance at end of month 9	Balance at end of month 10	Balance at end of month 11	Balance at end of month 12
First bank						
Second bank						
Third bank						

b) Based on the table, for how many months is it best to put our money in the first bank, and for how many months in the second bank?

(16 points)

8. An ambulance airplane carrying a severely injured person was heading to the hospital of the military base at $A(50, 240)$ (coordinates in km), but due to a technical problem over the open sea it had to make an emergency landing at the point $C(50, 20)$. The other ambulance airplane of the service was at $D(160, 170)$, heading to base A ; it had enough fuel for 160 km and carried no patient. What is the minimum time, counted from the emergency landing, in which the injured person can be taken to the hospital at base A by the other airplane, if fuel is available at base A and on a small island at $B(210, 190)$, and the average speed of the airplanes is 1000 km/h? Refueling (if needed) and moving the patient to the other airplane take 10 minutes each.

(16 points)

9. a) Truthful (who always tells the truth), Liar (who always lies), and Uncertain (of whom we can never know whether he is lying or not) ran a race, and there was no tie. A mathematician, who watched the race but did not know them, asked them the following questions at the end of the race.

He asked the one who finished first: Who finished second? The answer was: Truthful.

He asked the one who finished second: Who are you? The answer was: Uncertain.

He asked the one who finished third: Who finished before you? The answer was: Liar.

In what order did they finish, according to the mathematician?

b) Answer the following questions (A, B, C denote sets).

We know that $A \setminus B = C$. Is it true that $B \cup C = A$?

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(16 points)