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translated from: KoMaL, March 2007, pages 137-139

Pdf file: KoMaL, March 2007, pages 137-139

From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. Find the exact values of the following expressions.

$$a = \lg 1000^{2007} + \lg 0.001^{2007} \quad (2 \text{ points})$$

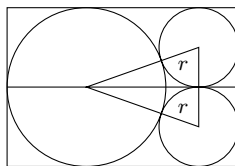
$$b = (2005! + 2006!) \left(\frac{1}{2006!} - \frac{1}{2007!} \right) \quad (4 \text{ points})$$

$$c = \left(\frac{2}{\sqrt{2007} - 1} \right) \left(\frac{1}{1 + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{5}} + \dots + \frac{1}{\sqrt{2005} + \sqrt{2007}} \right) \quad (5 \text{ points})$$

2. The diagonal of a lateral face of a right prism with a square base is 10 cm, and the space diagonal of the prism is 12.5 cm. Find the surface area and the volume of the prism.

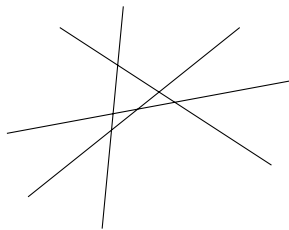
(12 points)

3. In a rectangular park, three flower beds in the shape of circles are made, touching each other and the sides of the rectangle as in the figure. Paths are built along the perimeter of the park, along the circles, and along all the lines through the centers of the circles drawn in the figure. The two smaller, equal flower beds have radius $r = 15$ m.



- a) What is the radius of the large circle? (2 points)
- b) What is the total length of the paths? (8 points)
- c) What percent of the area of the whole rectangle do the three circles cover? (4 points)
4. There are 10 lines in general position in the plane. (No two of them are parallel, and no three of them pass through the same point.)
- a) How many intersection points do the 10 lines have? (2 points)
- b) How many non-overlapping segments and how many rays can we count on the 10 lines? (5 points)
- c) We choose two of the resulting pieces (segments and rays) at random. What is the probability that the two pieces are of the same type (both segments or both rays)? (8 points)

(For example, for 4 lines in general position, as in the figure, we can count 6 intersection points, 8 segments, and 8 rays, that is, 16 pieces.)



Part II

5. Given the points $A(2, 2)$ and $B(9, 9)$, find the equation of the circle that passes through A and B and is tangent to the x -axis.

(16 points)

6. We write the odd natural numbers in the following triangular table so that each row contains one more number than the previous row:

			1		
		3		5	
	7		9		11
13		15		17	19
21	23	...			

- a) What number is at the beginning of the 20th row? (3 points)
- b) In terms of n , find the number at the beginning and at the end of the n th row, and the sum of the numbers in that row. (6 points)
- c) What is the sum of the numbers in the row that contains the number 2007? (3 points)
- d) We choose one of the first 100 rows at random. What is the probability that the sum of the numbers in the chosen row is a perfect square? (4 points)
7. a) Solve the equation $4^{x+1} - 13 \cdot 6^x + 9^{x+1} = 0$ over the real numbers. (7 points)
- b) Find all angles x in the interval $[0^\circ, 180^\circ]$ that satisfy the following equation.

$$2 \sin^2 x + 13 \sin x \cos x - 3 \cos^2 x = 6$$

(9 points)

8. Inscribe a rectangle in the acute triangle ABC of area T as in Figure 1.

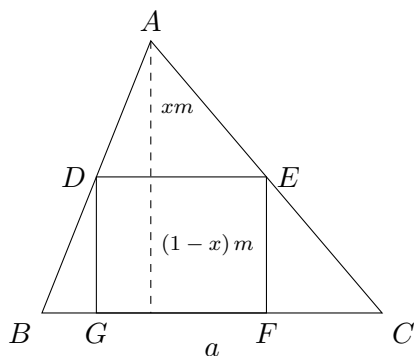


Figure 1

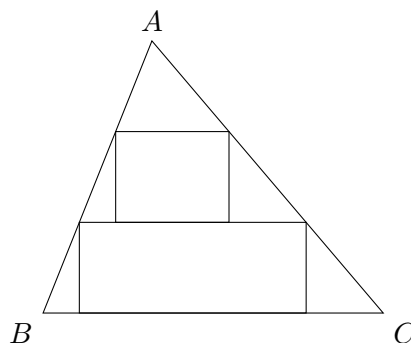


Figure 2

Advanced Level Practice Set - 2007 (March 2007)

- a) The height of the triangle ADE from A is x times the height m of the triangle ABC from A ($0 < x < 1$). Express the area of the triangle ADE and the area of the rectangle $DEFG$ in terms of T and x . (4 points)
- b) At most what fraction of the area of the triangle ABC can the rectangle cover? (5 points)
- c) At most what fraction of the area of the triangle ABC can the two rectangles inscribed as in Figure 2 cover? (7 points)
9. Find all natural numbers n that satisfy both of the following conditions.
- The decimal form of $\frac{1}{n}$ is terminating.
 - n^2 has three times as many positive divisors as n .

(16 points)