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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. Find all integers a, b, c with the following two properties:

- i) the sum of the three numbers is half of their product;
- ii) one of the numbers is equal to the sum of the other two.

(11 points)

2. The flight paths of two airplanes flying in the same plane are two perpendicular lines. At a given moment, one airplane is 2 km and the other is 2.5 km from the imaginary intersection point of their paths. The speed of the first airplane is 270 km/h, and that of the second is 180 km/h. The warning system of the airplanes warns the pilots if they get closer to each other than 1 km.

- a) Find after how many seconds the distance between the two airplanes is the smallest.

(9 points)

- b) Will the warning system go off on the airplanes?

(3 points)

3. If we swap the second and third of the first three terms of a non-constant arithmetic sequence, we get the first three terms of a geometric sequence. The first term of the arithmetic sequence is equal to the opposite of the product of its second and third terms. Find the arithmetic sequence.

(14 points)

4. In how many different ways can we form from a class of 30 students

- a) a group of five?

(2 points)

- b) a group of at most five, but at least two students?

(4 points)

- c) a group of five, if the president of the class student council must be in it?

(4 points)

- d) a group of five, one member of which is chosen as the group leader?

(4 points)

Part II

5. A vessel in the shape of a right circular cone has a base radius of 12 cm and a height of 18 cm. We stand the vessel on its vertex and pour 1 liter of water into it.

- a) How high does the water stand in it?

(8 points)

- b) How much paint would be needed to paint the outer wall of the vessel if 1 liter of paint covers 8 m^2 ?

(8 points)

6. In the traditional Hungarian lottery (5 numbers out of 90), the chance of winning with one ticket (at least two matches) is about 2.33%. Hoping for the big prize, we send 100 randomly filled out tickets.
- What is the probability that we will have at least one winning ticket? (5 points)
 - What is the probability that we will have exactly three winning tickets? (5 points)
 - How many tickets would we have to play in a given week so that with probability at least 99% there is a winning ticket among them? (6 points)
7. We want to determine the height of a radio tower standing on top of a mountain that we cannot get to. On the plain at the level of the foot of the mountain we mark points A and B , 80 meters apart, so that from B the point A and the tower are seen in the same direction (in this order). From A the foot of the tower is seen at an angle of elevation of $41^\circ 30'$ and its top at 49° ; from B the foot of the tower is seen at $34^\circ 15'$.
- How high is the mountain? (7 points)
 - How high is the tower? (5 points)
 - At what angle of elevation is the top of the tower seen from B ? (4 points)
8. In the coordinate plane, the circle k_1 is given by the equation

$$x^2 + y^2 - 4x - 2y - 20 = 0$$

- A right triangle is inscribed in the circle k_1 ; two of its vertices are $(-2, 4)$ and $(5, 5)$. Find the coordinates of the third vertex. (6 points)
 - Find a point on the x -axis from which the tangent segments drawn to k_1 and to the circle $k_2: (x - 12)^2 + y^2 = 4$ have equal lengths. (10 points)
9. a) Find the area of the region bounded by the graph of the function $f(x) = px^2 + 2x + q$ and the lines $x = 4$ and $x = 6$ (and the x -axis), if

$$\int_0^2 f(x) dx = -2 \quad \text{and} \quad \int_2^4 f(x) dx = 54$$

(9 points)

- b) Find the values of k for which

$$\int_0^k f(x) dx = 0$$

(7 points)