

Advanced Level Practice Set- 2007 (September 2007)

author: Mrs. Sándor Szlovák

translated from: KoMaL, September 2007, pages 330-331

Pdf file: KoMaL, September 2007, pages 330-331

From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. a) An urn contains 100 slips numbered from 1 to 100. We draw two slips at the same time. What is the probability that the numbers on the slips are twin primes? (Twin primes are two primes that are consecutive odd numbers.)
b) What is the probability of drawing “triplet primes” if we draw three slips at the same time?

(11 points)

2. Is the binomial coefficient $\binom{38}{19}$ divisible by 17?

(12 points)

3. In the equation of the circle

$$x^2 + y^2 + kx - \frac{k^2}{2}y + 1 = 0$$

find the value of the parameter k so that the circle is tangent to both coordinate axes.

(14 points)

4. a) Solve the equation $[x]^2 + 3 \cdot [x] - 10 = 0$ over the real numbers. ($[x]$ denotes the greatest integer not greater than x .)
b) For which integers p and q is the solution set of the equation $[x]^2 + p[x] + q = 0$ a single interval of length one?

(14 points)

Part II

5. Find the values of the real parameter a for which the real roots of the equation

$$x^3 + (3 - 2a)x^2 - a^2x - 3a^2 + 2a^3 = 0$$

form a geometric sequence in some order.

(16 points)

6. In a regular polygon, A , B , C , and D are four consecutive vertices, and the length of the segment AC is the quadratic mean (root mean square) of the lengths AB and AD . How many sides does the polygon have?

(16 points)

7. a) Solve the equation

$$2x^2 - \left[2\sqrt{2} \sin \left(\alpha + \frac{\pi}{4} \right) \right] x + \sin 2\alpha = 0$$

where α is a real parameter.

- b) Show that the function $f(\alpha) = \sin^4 \alpha + \cos^4 \alpha$, defined on the real numbers, is a transformation of the function $g(\alpha) = \cos^2 \alpha$, defined on the real numbers.

(16 points)

8. a) Show that every member of the family of curves $y = x^3 + ax^2 + (2a - 3)x + a - 2$, where a is an arbitrary real number, passes through one point. Find the coordinates of this fixed point.

- b) How should the value of the parameter a be chosen so that the tangent line of the corresponding curve at $x_0 = 2$ passes through the point $P(-1, 0)$?

(16 points)

9. a) Find the value of the parameter a if

$$\lim_{x \rightarrow \infty} f \left(\frac{2^x - 1}{2^x + 1} \right) = 0$$

where $f(x) = ax^2 - 3x + 2$.

- b) Find the area of the region bounded by the graph of the function $f(x) = x^2 - 3x + 2$ and the x -axis.

(16 points)