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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. Solve the following equations.

a) $\frac{9}{x^2 - 16} + \frac{5}{4 - x} + \frac{13 + x}{x + 4} = -\frac{18}{4 + x}$

b) $2 \cos^2 x - 3 \sin x = 0$

(11 points)

2. We draw 2 cards from a 32-card Hungarian deck (four suits: hearts, bells, leaves, and acorns, with 8 cards each). The order of drawing does not matter.

- a) How many different pairs of cards can we get?
- b) How many pairs of cards contain an acorn?
- c) In how many different orders can the hearts be in the full deck?
- d) What is the probability that in the shuffled full deck there is no heart in the first two places?

(12 points)

3. At a company, the bonus was to be divided among six people in the ratio 1 : 2 : 3 : 4 : 5 : 5. Meanwhile it turned out that one of the employees, who would have received 25% of the money, did not meet the requirements. So they wanted to divide the 225 000 forints intended for him among the other five people so that the ratio of the amounts they receive does not change. How much money does each of the five people get?

(14 points)

4. The vertex A of a square with side 10 units is the origin, and the slope of one of its sides is 2. The square is above the x -axis.

- a) Find the exact coordinates of the vertices of the square.
- b) We reflect the square in the x -axis. What are the coordinates of the vertices of the reflected square?
- c) What is the area of the smallest rectangle with sides parallel to the coordinate axes that contains both squares?

(14 points)

Part II

5. A circle with center O and radius 5 cm is given. Let C be the point on the extension of the diameter AB beyond B , 5 cm from B . From this point we draw a tangent line to the circle; let the point of tangency be E .

- a) Prove that the triangle ACE is isosceles.
- b) Find the side lengths and the angles of the triangle ACE .

- c) What is the area of the triangle ACE ?
- d) Find the exact value of the radius of the circle inscribed in the triangle ACE .

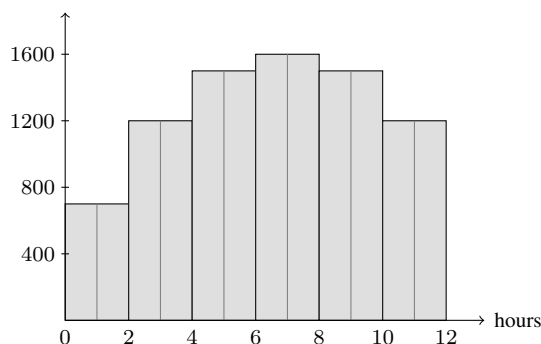
(16 points)

6. a) Solve the equation $2 \cdot \left(\frac{1}{4}\right)^{3x} - \left(\frac{1}{8}\right)^{x-1} = 384$.

- b) Does the equation $2 \log_{\frac{1}{4}} 3x - \log_{\frac{1}{4}} (x-1) = a$ have a real solution if the parameter a is a nonnegative real number?

(16 points)

7. The figure shows the result of a traffic count on a road section from midnight to noon. On the horizontal axis we show the hours, and on the vertical axis the number of passing cars, rounded to hundreds.



The traffic counter added up the vehicles passing there in two-hour periods and showed the data averaged over one hour each on a bar chart. (The area of the bars gives the number of cars passing in the given time interval.) After drawing the chart, he noticed that the six midpoints of the upper horizontal sides of the bars lie on a parabola (see the figure in the solution). According to him, this parabola shows the change of the traffic on the given road section more realistically.

- a) According to the chart, how many cars passed on the given road section in the given time?
- b) If we approximate the number of cars not in this way, but by the area under the parabola, what result do we get?

(16 points)

8. In a mathematics competition 30 problems were given. András solved 16 problems correctly, and Balázs 14. It turned out that there were 7 problems that both of them solved correctly.

- a) How many problems did neither of them solve?
- b) Dénes, who also took part in the competition, found that there were 5 problems that all three of them solved correctly. Dénes compared his solutions with András's and then with Balázs's, and they found that he had 8 correct solutions in common with András and 9 with Balázs. Finally they also noticed that every problem was solved correctly by at least one of them. Who among them had the most correct solutions?
- c) We choose one of the 30 problems at random. What is the probability that exactly two of them solved it correctly?

(16 points)

9. If both roots of the equation $x^2 + (3p-2)x - (7p+2) = 0$ are nonnegative, consider the two numbers as the edge lengths of two cubes. Find the value of the parameter p for which the sum of the surface areas is minimal.

(16 points)