Advanced Level Practice Set - 2008 (December 2008)

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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.
The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.
This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. Uncle Toni sold pogácsa (savory scones) and pretzels at the sports field in Pécel. He made a 25% profit on the pogácsa and a 60% profit on the pretzels. One day he sold the same amount (in value) of pogácsa as of pretzels, and so he made a 50% profit. The next day, however, he sold twice as much pogácsa as the day before, and the same amount of pretzels.

   a) What percent was his profit on that day? *(8 points)*

   b) Later Uncle Toni added gummy candy and muffins to his offer. Endre buys three items to cheer with. In how many ways can he do this if he may buy more than one of the same product? *(4 points)*

2. A month on the planet U2CK3 has 41 days. According to the locals, every day is suitable for at least one of the three noble activities (flying, studying, cooking). In a month only three days are suitable for all three noble activities. The number of days suitable for flying is 19, for studying 23, and for cooking 19.

   a) The numbers of days of the month that are suitable only for flying and cooking, only for flying and studying, and only for studying and cooking, are three consecutive terms of a geometric sequence with ratio 2. How many days of the month are suitable for exactly one noble activity? *(6 points)*

   b) If we choose three of the 41 days at random, what is the probability that all three days are suitable only for cooking? *(6 points)*

3. a) Evaluate the following integral.

$$\int \frac{1}{\cos^2 x \cdot \sin^2 x}\, dx$$

   (8 points)

   b) Find the derivative of the function $f(x) = \left(x^2 + 3\right)\left(x^3 + 2x - 1\right)$. *(5 points)*

4. a) Graph the following function.

$$f(x) = \begin{cases} -x^2 + 4, & \text{if } x \geq 0 \\ \dfrac{x^2 - 5x}{x^2 - x - 20}, & \text{if } x < 0 \text{ and } x \neq -4 \end{cases}$$

   (8 points)

   b) Let k be a real number. How many zeros does the function $g(x) = f(x) - k$ have? *(6 points)*

   Note: KoMaL prints the condition as "$x < 0$ and $x \neq 4$"; the excluded value is $x = -4$.

Part II

5. A civil engineer has to design and build a fountain. The lot is bounded by a wall on the west and by a hedge on the south; the wall and the hedge are perpendicular to each other. On the lot there is a plum tree 7 m from both the wall and the hedge, a cherry tree 5 m from the wall and 3 m from the hedge, and a statue 18 m from the wall and 9 m from the hedge. The fountain has to be placed so that it is the same distance from the two trees, and it is twice as far from the statue as from the cherry tree.

   a) How far from the statue will the fountain be built? *(11 points)*

   b) Ede would do the earthwork for the fountain in 10 hours, Béla in 12 hours. Béla starts at 7 in the morning, Ede joins him an hour later, they take a half-hour break once, and then they work together until the job is done. At what time do they finish? *(5 points)*

6. In an arithmetic sequence the sum of the reciprocals of the first and the fourth terms is 5.5. The first, second, and sixth terms of the sequence are three consecutive terms of a geometric sequence. Find the first term and the difference of the arithmetic sequence.

   (16 points)

7. We roll two dice. Let X be the absolute value of the difference of the two numbers rolled.

   a) What is the probability that X is a perfect square? *(6 points)*

   b) Graph the distribution of the random variable X. *(5 points)*

   c) Find the expected value of X. *(5 points)*

8. The sides of the triangle ABC are $a = 13$ cm, $b = 14$ cm, $c = 15$ cm.

   a) Find the length of the median s_a from the vertex A and of the angle bisector f_c to the side c. *(8 points)*

   b) Find the radii of the inscribed and the circumscribed circles of the triangle. *(8 points)*

9. A container in the shape of a right prism whose base is an isosceles trapezoid holds some water. The parallel sides of the trapezoid are 4 cm and 10 cm, its legs are 5 cm, and the length of the prism is 11.2 cm. If we turn the container from its trapezoid-shaped base onto its largest lateral face, the height of the water becomes one third of what it was. Find the volume of the water in the container.

   (16 points)

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