Advanced Level Practice Set - 2008 (February 2008)

author: József Czinki
translated from: KoMaL, February 2008, pages 78-79
Pdf file: KoMaL, February 2008, pages 78-79

From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.
The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.
This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. We poured together 5 kg of 15 percent, 8 kg of 20 percent, and 12 kg of 40 percent (by mass) sugar solution. The solvent was water in each case. Since then, 40 dkg (0.4 kg) of sugar fell into it, 12 percent of the water evaporated, and the mixture is uniform.

   a) What is the concentration of the solution now (percent by mass)?

   b) What fraction of the solution has to be replaced by water if we want to get a 10 percent solution?

   (11 points)

2. The following table shows the populations of the continents in 1995.

Continent	Population (millions)
Europe	682
Asia	3498
Africa	730
North America	293
Central and South America	481
Australia and Oceania	32

   a) Find the mean and the standard deviation of the data.

   b) Make a diagram of the distribution of the Earth's population by continent.

   (12 points)

3. a) We know that p and $10p - 1$ are positive primes. Can $10p + 1$ be a prime?

   b) Péter and Pál are both very clever. Once Péter said to Pál:

   – Three people just visited me. The sum of their ages is 4 more than your age, the product of their ages is twice the square of 35, and each of them is older than four.

   – I do not know the answer, said Pál after a short thought.

   – One of my visitors is older than me, Péter continued the conversation.

   – Then I know the answer, said Pál.

   What is the difference between the ages of Péter and Pál?

   (14 points)

4. a) We are given 2007 points in the plane, no three of which are on a line. Three of these points are green. Consider all triangles determined by the 2007 points. How many of them have a green vertex?

   b) Prove that any two neighboring terms of the Fibonacci sequence are relatively prime. (The rule of the Fibonacci sequence: $a_1 = 1$, $a_2 = 1$, $a_n = a_{n-1} + a_{n-2}$ for every $n > 2$.)

(14 points)

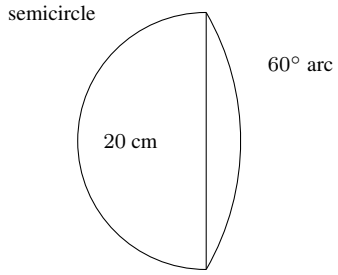
Part II

5. a) Solve the equation $2x^3 - 3x^2 + 3x - 1 = 0$.

   b) How many solutions does the equation $\cos x = \dfrac{|x|}{2\pi}$ have?

(16 points)

6. The figure shows the cross section of wooden fence posts. Each of the 80 fence posts is 3 meters long, and 1 meter of each will be in the ground. This part of the fence posts is soaked with a special liquid to make them last longer.



   a) How much liquid is needed if 1 m^3 of wood "absorbs" 2.5 liters of liquid?

   b) How much paint is needed to paint the fence posts after they are set up, if 1 kg of the paint covers 4 m^2?

(16 points)

7. Shipwrecked people on an uninhabited island without vegetation plan to build a new, smaller boat from the pieces of their original sailboat, which ran aground in the storm. The storm broke the mast at random into three pieces. If a piece at least 20 m long remained of the original 40 m long mast, the boat can be built. What is the probability that when they swim back to the wreck, they find such a piece?

(16 points)

8. A company ordered 100 closed containers in the shape of a cylinder, each with an inner volume of 1 m^3, made of special sheet metal. The customer specified only the volume and the shape. What should the height of the cylinder be if the company wants to use as little material as possible?

(16 points)

9. A commander examines, from the top of a 480 m high hill, the position of his own artillery, stationed on flat terrain along the seashore, and of an enemy infantry corps, also on flat terrain along the seashore. The apparent distance of the two corps (the angle between the lines of sight) is 71.2°. The commander sees the artillery at an angle of depression of $6°30'$, and the infantry at an angle of depression of 8°. Will the commander, who is good at mathematics, give the order to fire if he knows that the maximum range of his artillery's cannons is 4 km?

(16 points)

 Downloaded from martahidegkuti.com