

Advanced Level Practice Set - 2008 (March 2008)

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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. How many vertices can a simple complete graph have if the number of its edges is less than six times, but more than five times the number of its vertices?

(11 points)

2. Solve the following system of equations over the pairs of real numbers.

$$\begin{aligned}4^{\frac{x+y}{1004}} - 4^{\frac{x+y}{2008}} &= 12 \\ 2007^{\log_x(2008y-x)} &= 1\end{aligned}$$

(12 points)

3. Find the points (x, y) of the coordinate plane whose coordinates satisfy

$$\begin{aligned}\cos(x+y) &= 1 \\ \sin(x-y) &= 0\end{aligned}$$

(14 points)

4. A 24-person guesthouse has one 4-bed room, one 8-bed room, and one 12-bed room. In the 4-bed room the price is 1000 forints per person per night, in the 8-bed room 800 forints, and in the 12-bed room 700 forints. To save money, the rooms are heated depending on the number of guests, as shown in the table:

number of people	1–4	5–8	9–12	13–16	17–20	21–24
heated rooms	4-bed	8-bed	12-bed	4-bed + 12-bed	8-bed + 12-bed	all three

If more than one room is heated, the cheaper places can be filled first. A group of any size from 1 to 24 people can rent the house.

- a) For which group is the average price per person lower: a group of 15 or a group of 21 people renting the guesthouse?
b) Is there a group size for which it is worth renting the house for more people than there are in the group?
c) Order the group sizes in decreasing order of the average price per person.

(14 points)

Part II

5. A bag contains blue and red balls. The probability of drawing two blue balls in two draws is one fifth of the probability of drawing two red balls, and one sixth of the probability of drawing one ball of each color. Each ball is equally likely to be drawn. How many red and how many blue balls are in the bag?

(16 points)

6. The differences of the corresponding terms of a geometric sequence and an arithmetic sequence are 0, -3 , 10, 103. Find the two sequences.

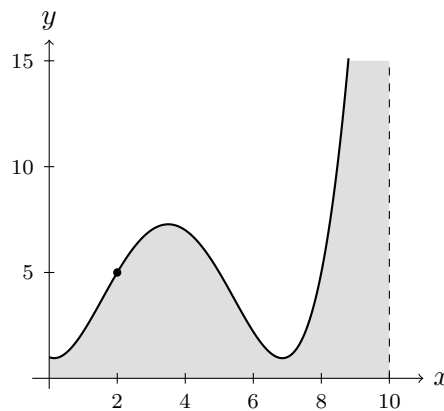
(16 points)

7. Find the equation of the line e through the point $(2, 5)$ of the function

$$f(x) = \frac{1}{20} (x+1)(x-2)(x-5)(x-8) + 5$$

such that on the interval $[0, 10]$ half of the area enclosed by the two curves is above the line e , and half is below it.

(16 points)



8. From a group of people, k people can be chosen in p ways, $k+1$ people in $2p$ ways, and $k+2$ people in $3p$ ways. How many people are in the group?

(16 points)

9. The shadow of the Earth falls on the disk of the Moon so that the common chord of the two circles is seen at an angle of 120° from the center of the Moon's disk, and at an angle of 60° from the center of the Earth's shadow. What fraction of the Moon's disk is not visible?

(16 points)