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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. Solve the following equation over the real numbers.

$$(x - 2)^2 + \sqrt{4 - 4x + x^2} + \frac{6x - 18}{3 - x} = 0$$

(11 points)

2. One of the most precious, strictly protected rare plants of the Aggtelek National Park is the Tornal wallflower (*Onosma tornensis*). Its notional value is 250 000 forints.

A researcher studying the plant found that the growth of the wallflower is approximately described by the function $h(x) = 2 + \log_2(x + 1)$, where h is the height of the plant in cm, and x is the time elapsed in days, $x \in [0, 7]$.

- a) How tall is the plant on the last day of the measurement? (3 points)
 - b) On which day does its height reach 4 cm? (3 points)
 - c) The population is still significant in Hungary. About 30% of the population in the European Union is found here, about 600 000 plants. With a decrease of 4% per year, how long does it take for the Hungarian population to decrease to one quarter? (6 points)
3. The hypotenuse of a right triangle is 5 units long, and the sines of its angles, in increasing order, are three consecutive terms of an arithmetic sequence. What is the area of the triangle? (14 points)
 4. The two altitudes drawn from the vertex of one of the obtuse angles of a rhombus are 13 units long, and the segment joining their feet is 10 units long.
 - a) What are the angles of the rhombus? (10 points)
 - b) What is the perimeter of the rhombus? (4 points)

Part II

5.
 - a) How many four-digit numbers can be formed from the even digits, if the digits cannot be repeated? (3 points)
 - b) What is the sum of the four-digit numbers obtained in this way? (5 points)
 - c) We write the even digits on five slips, put them into an urn, draw four of them, and place them next to each other in the order of drawing. If the resulting number has four digits, what is the probability that it is divisible by six? (8 points)

6. The coordinates of two adjacent vertices of a parallelogram are $A(8, 9)$ and $B(0, 3)$, and the equation of the line of one of its sides is $x + 2y = 6$. The area of the parallelogram is 80. Find the coordinates of the missing vertices.

(16 points)

7. The function

$$f(x) = \frac{2x^2 - 9x - 11}{x^2 - 5x - 6}$$

is defined on the real numbers $\mathbb{R} \setminus \{-1, 6\}$.

- a) Graph the function f . (8 points)
- b) Find the smallest and the largest value of f on the interval $[0, 5]$. (2 points)
- c) Find the equation of the tangent line of the graph of f at the point with x -coordinate 5. (6 points)
8. We cut out of paper a square with side 4 dm and a regular triangle with side 4 dm, and then we cut the square along its diagonals into four congruent right triangles. From three isosceles right triangles and the equilateral triangle we can assemble the net of a pyramid, which is a 1 : 20 scale model of a building (considering the edges).
- a) What is the surface area of the building, including the base? (9 points)
- b) We want to make the solid building out of concrete. How much of each material do we use, if 10 m³ of concrete needs 2500 kg of cement, 12 m³ of gravel, and 2400 liters of water? (5 points)
- c) What is the material cost of the concrete used, if from ordering to delivery (and payment) the material prices rose by 15%, and at the time of planning 1 quintal (100 kg) of cement cost 3100 forints and 1 m³ of gravel cost 4200 forints? (The water was taken from our own well, so it did not increase the costs.) (3 points)
9. The quadratic function $f(x) = x^2 + 2x + p^3 + 3p^2 + 2p$ has two different zeros. Find the value of the parameter p for which the product of the zeros equals the square of the value of f at 0.

(16 points)