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Pdf file: KoMaL, October 2008, page 402

From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. Solve the following equation over the real numbers.

$$\frac{\log_2(2x - 5)}{\log_2(x^2 - 8)} = \frac{1}{2}$$

(12 points)

2. What is the area of the isosceles trapezoid circumscribed about a circle of radius 3 cm, whose acute angles are 60° ?

(12 points)

3. During parachutists' target jumps, it was recorded at how many meters from the center Berci landed in nine of his ten jumps. The recorded data in meters: 1, 3, 5, 2, 4, 6, 8, 9, 7. The mean absolute deviation of the data of his ten jumps from 5 m was 2.4 m.

a) Find the missing tenth data value.

b) Find the mean, the mode, the median, and the standard deviation of the data of the ten jumps.

(13 points)

4. How many zeros are at the end of the number $\binom{100}{50}$?

(14 points)

Part II

5. The opposite edges of a tetrahedron are perpendicular to each other. Show that there is a sphere that passes through the midpoints of all six edges.

(16 points)

6. Emese's monthly net salary is 150 000 forints. Assume that her net monthly salary is raised by 10% every year. After how many years can she buy an apartment worth 15 000 000 forints from her salary, if she saves 60% of her salary every month and the price of the apartment does not change meanwhile?

(16 points)

7. Two curves are given by the equations $y = \sqrt{9 - x^2}$ and $y = x^2 - 3$. Find the volume of the solid obtained by rotating the region bounded by the two curves about the y -axis.

(16 points)

Advanced Level Practice Set - 2008 (October 2008)

8. Three companies deliver concrete to a construction site. The first has 5, the second 4, and the third 6 concrete trucks. On a given day the construction needs 12 truckloads of concrete. How many trucks should be ordered from each company so that the delivery cost is minimal, if the delivery cost per truck is 40 000, 60 000, and 50 000 forints for the three companies?

(16 points)

9. The quadratic equation $ax^2 + bx + c = 0$ has real roots. We know that $|a + b + c| < |a|$. Prove that at least one root of the quadratic equation is in the interval $(0, 2)$.

(16 points)