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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. Solve the following equations over the real numbers.

a) $\frac{x+3}{3} = \frac{x^2+3x-4}{2x^2+4x-16} \cdot (x+3)$

b) $\lg(6^x - 96) - 2 = \lg 2 + \lg 6$

(11 points)

2. On a subway escalator, a bag gets down to the subway level in a seconds. A passenger walks the same way on the escalator when it is not working in b seconds. How long does it take the passenger to get down to the subway level on the working escalator if he walks with the same steps as when it is not working?

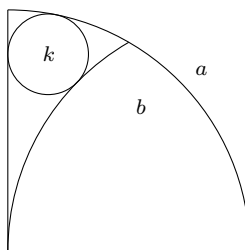
(12 points)

3. Solve the following system of equations over the real numbers.

$$x + y = \frac{1}{3}, \quad \cos \pi x \cdot \cos \pi y = \frac{1}{2}$$

(14 points)

4. From one endpoint of the arc of a quarter circle of radius r (the arc a in the figure), as center, we draw an arc also of radius r (the arc b), which divides the circular sector determined by the quarter circle into two parts.



Find the radius of the circle k inscribed in the smaller part.

(14 points)

Part II

5. Two vertices of a triangle are $A(6, 10)$ and $B(2, 7)$, and the third vertex is on the line $y = 2x + 3$. Find this vertex so that

- a) the triangle is isosceles with base AB ;
b) the sum of the squares of its sides is minimal.

(16 points)

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6. a) The line $x = p$ bisects the area of the region bounded by the parabolas $y = -x^2 + 6x$ and $y = x^2 - 4x + 8$. Find the value of p .
- b) We rotate the region bounded by the parabolas $y = -x^2 + 6x$ and $y = x^2 - 4x + 8$ about the x -axis. Find the volume of the resulting solid.

(16 points)

7. In the Csoki Factory, a consumer protection inspection found that a bag contains exactly the prescribed (standard) 40 pieces of candy with probability 0.95, and more or fewer with probability only 0.05.
- a) We choose 5 bags at random. What is the probability that each of them contains exactly 40 pieces of candy?
- b) What is the probability that among the five bags we find at least two that are not standard?
- c) What is the probability that exactly 95 of 100 bags are standard?

(16 points)

8. A company supports a university with 10 million forints. The money is deposited in a bank at 7% annual interest. According to the conditions, the amount must reach 15 million forints, and then in the following 20 years, at the beginning of each year, it must be divided equally among the 10 best first-year students as a scholarship, so that the money runs out with the last payment. The amount earns interest all along, and the annual interest is 7% throughout.
- a) After how many years do they start paying the scholarships?
- b) How much money does each student receive?

(16 points)

9. The towns A , B , and C , at the same height above sea level, form a triangle: A and B are 53 km apart, B and C are 45 km apart, and A and C are 28 km apart. At the point dividing the segment AB into three equal parts that is closer to A , an 800 m high storm warning tower was built.
- a) At what angle of elevation is the top of the tower seen from the town A ?
- b) How far (in a straight line) is the top of the tower from the town D , which is 20 km from both A and C ?

(16 points)