

Advanced Level Practice Set - 2009 (January 2009)

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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. A cyclist rode the first half of his trip at 12 km/h and the second half at 18 km/h. On the way back he would like to ride at a constant speed and cover the distance in the same time as on the way there. What should his speed be?

(11 points)

2. Solve the following system of equations over the pairs of integers.

$$\begin{aligned}x^2 - 4y^2 - 7x + 14y &= 0 \\ xy + x + 2y &= 0\end{aligned}$$

(13 points)

3. We choose a point C on the segment AB such that $AC = x$ and $BC = y$. On the same side of the segment AB we draw the squares $ACDE$ and $CBHG$. The line DC meets the line BE at P , and the line AH at Q . Express the lengths CQ and CP in terms of x and y .

(13 points)

4. How many terms of the sequence

$$a_n = -1 + \lg(n + 3)$$

are greater than 1 but less than 2?

(14 points)

Part II

5. In a coordinate system the points $A(-1, 4)$ and $B(5, 2)$ are given. Find

a) the point P on the x -axis with $PA = PB$, and the point Q on the y -axis with $QA = QB$; (5 points)

b) the point R on the x -axis for which $AR + RB$ is minimal; (5 points)

c) the point C on the x -axis for which $AC^2 + CB^2$ is minimal. (6 points)

6. a) We roll two fair dice, a yellow one and an orange one. Let c be the number on the yellow die and n the number on the orange die. What is the probability that the equation $cx^2 = n$ has two integer roots? (8 points)

b) We roll three fair dice. Let p be the number on the red die, f the number on the white die, and z the number on the green die. What is the probability that the equation $px^2 + fx + z = 0$ has two different roots that are reciprocals of each other? (8 points)

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7. a) Show that every member of the family of curves $y = x^3 + (a - 3)x^2 - 3x + 2$ passes through one point. Find the coordinates of this fixed point. *(6 points)*
b) How should the parameter a be chosen so that the tangent line of the corresponding curve at $x_0 = 3$ passes through the point $(0, 2)$? *(10 points)*
8. The inscribed circle of a right triangle has radius 2, and the radii of the excircles tangent to the legs are 3 and 10. What is the radius of the excircle tangent to the hypotenuse? *(16 points)*
9. The 12 lattice points (points with integer coordinates) on the circle of radius 13 centered at the origin determine a 12-gon.
a) Is this 12-gon a tangential polygon (does it have an inscribed circle)? *(5 points)*
b) Find the area of the 12-gon. *(7 points)*
c) This 12-gon is the base of a right pyramid of height 13. What angles do the lateral faces make with the base? *(4 points)*