

Advanced Level Practice Set - 2009 (March 2009)

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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. Solve the following inequality over the real numbers.

$$27^x < 6 \cdot 9^x + 3^{x+3}$$

(12 points)

2. The longest side of a triangle is 16 and the shortest side is 10. The largest angle of the triangle is twice the smallest angle. Find the angles and the missing side of the triangle.

(12 points)

3. Anna and Bence roll four fair dice in a game. Anna wins if there are equal numbers among those rolled, and Bence wins if there is at least one 6 among them. If both conditions hold, the game is a draw; if neither holds, they keep playing.

a) Which of them has the greater chance of winning?

(7 points)

b) What is the probability that a game ends in a draw?

(6 points)

4. The sides of the triangle ABC are $AB = 34$, $BC = 16$, $AC = 30$. Let P be the center of the circumscribed circle and Q the center of the inscribed circle. Find the length of the segment PQ .

(14 points)

Part II

5. Solve the following equation over the pairs of real numbers.

$$\sqrt{4 - 4x^2 + 4xy - y^2} = \log_x(y - 2) + \log_{y-2} x$$

(16 points)

6. We drill a hole through a cube of edge a , perpendicular to one of its faces. The removed part is a square prism whose base is a square of side b and whose height is a .

a) Prove that if the volume of the resulting solid is 25% less than the volume of the original cube, then its surface area is 25% more than the surface area of the original cube.

b) Prove that no other number can be written in place of 25 in this statement, that is, a $k\%$ decrease in volume cannot come with a $k\%$ increase in surface area for any a and b , if $k \neq 25$.

(16 points)

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7. 30 students of a class wrote a math test. The mode of the grades is 2, the median 3.5, the range 4, and the average 3.2. We also know that there was one more 4 than 5, and the standard deviation of the grades is less than 1.36. Find the frequencies of the grades. (The grades are 1, 2, 3, 4, 5.)

(16 points)

8. The equation of the line e is $2x - y = 2$, and the equation of the parabola P is $y = x^2 + 8x + 17$. The points A and B lie on the line e , and the point C on the parabola P . The x -coordinate of A is -1 , and the y -coordinate of B is 2.

Find the coordinates of C so that the area of the triangle ABC is as small as possible. What is this minimal area?

(16 points)

9. Three two-digit natural numbers are three consecutive terms of an increasing arithmetic sequence, and their sum is 198. Writing the three numbers one after the other in increasing order, the resulting 6-digit number is divisible by 2009. Find the three numbers.

(16 points)