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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. The sides of a rectangular fish pond are 120 m and 160 m long. A pole was driven in along one diagonal of the rectangle, at a point from which the two endpoints of a shorter side are seen at a right angle. How far is the pole from the sides of the pond?

(11 points)

2. Solve the following inequalities over the real numbers.

a) $\sqrt{x^2 - 5x + 4} > -1$

b) $\log_{\frac{1}{2}}(4^x - 5 \cdot 2^x + 8) < -2$

(12 points)

3. The chess players of two schools played a tournament. Everyone played everyone else once. First the games within each school were played, 36 games in all. When the students of the two schools played each other, there were 42 games. How many students from each school took part in the tournament?

(14 points)

4. At the beginning of 2005 Antal deposited 100 000 Ft in a bank at 20% annual interest. Starting in 2005, at the beginning of every year Béla deposited b forints, also at 20% annual interest. At the end of 2009, Antal's and Béla's deposits had grown to the same value (from 2005 to 2009 neither of them took any money out). What is the value of b , rounded to 1000 Ft?

(14 points)

Part II

5. The axis of symmetry of the kite $ABCD$ is the diagonal AC , where $A(0, 0)$ and $C(8, 10)$. The area of the kite is 41 square units. One diagonal divides the other in the ratio 3 : 2, counting from the origin. Find the coordinates of the missing vertices.

(16 points)

6. We rotate an isosceles triangle about one of its legs, and then about its base. Let V_1 and V_2 denote the volumes of the resulting solids of revolution. Find the angles of the triangle if $V_1 : V_2 = 3 : 7$.

(16 points)

7. a) Find the digits a, b, c for which the one-digit $\overline{a}$, the two-digit $\overline{ba}$, and the three-digit $\overline{cba}$ positive numbers are consecutive terms of a geometric sequence.

- b) Find the digits a, b, c for which the one-digit $\overline{a}$, twice the two-digit $\overline{ba}$, and the three-digit $\overline{cba}$ positive numbers are consecutive terms of an arithmetic sequence.

(16 points)

8. A shooter hits the target with probability $\frac{1}{4}$.

a) What is the probability that out of 7 shots at least 2 hit the target?

b) At least how many shots does he have to take to hit the target with probability greater than $\frac{2}{3}$?

(16 points)

9. Graph the function

$$f(x) = 3^{1+\log_3[\cos(x+\frac{\pi}{4})]}$$

on the interval $\left(-\frac{3\pi}{4}, \frac{9\pi}{4}\right]$. Find the domain, the range, the zeros, the intervals of increase and decrease, and the period of f .

(16 points)