Advanced Level Practice Set - 2009 (September 2009)

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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.
The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.
This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. Show that for the function $f(x) = \dfrac{1}{x^2}$ (where $x \neq 0$ is a real number), for every pair of real numbers $a \neq 0$, $b \neq 0$ (with $a + b \neq 0$),

$$f(a) + f(b) + 2ab \cdot f(ab) = \frac{f(ab)}{f(a+b)}$$

(11 points)

2. Starting from some natural number, we choose every 12th number of the sequence of natural numbers until the sum of the chosen numbers is 2010. More than 10 but fewer than 100 numbers were chosen. Which number did we choose first, and how many numbers did we choose in all?

(12 points)

3. Show that if the sides of a triangle satisfy $a^2 + b^2 > 5c^2$, then c is the shortest side of the triangle.

(14 points)

4. Does the equation $\sin^6 x + \sin^4 x + 2\sin^2 x + \cos^2 x = 4\sin^5 x$ have a solution on the interval $\left(0, \dfrac{\pi}{2}\right)$?

(14 points)

Part II

5. Solve the following inequality over the real numbers.

$$x^2 \cdot \log_4\left(5x^2 - 2x - 3\right) - x \cdot \log_{\frac{1}{4}}\left(5x^2 - 2x - 3\right) \leq x^2 + x$$

(16 points)

6. Márton hits the upper hole of the "kicking wall" in the Stadium park with probability 10%, and the lower hole with probability 40%.

   a) Shooting once at each hole, what is the probability that he A: never hits; B: hits exactly once?

   b) How many times must he shoot at the lower hole to hit at least once with probability greater than 95%?

   c) What is the probability that, shooting three times at the lower hole and then three times at the upper hole, he hits exactly once?

   d) Krisztián noticed that Márton's hit probability is not constant. After a hit it increases by $\frac{1}{10}$ of the original probability, and after a miss it decreases by $\frac{1}{10}$ of the original probability. Márton shoots twice at the lower hole. What is the probability that he hits exactly once?

(16 points)

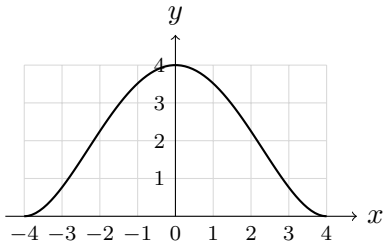
7. In a coordinate system, the circle with center $K\,(-2,3)$ and radius $r=5$ and three points are given: $A\,(6,2)$, $B\,(-3,5)$, and $C\,(-5,y_0)$, where $y_0>0$.

   a) What is the length of the chord of the circle on the line AB?

   b) The point C lies on the circle. Let e be the tangent line of the circle at C. What is the area of the triangle enclosed by the line e, the y-axis, and the line AB?

   (16 points)

8. The gable of a house waiting for renovation is 8 m wide and 4 m high (see the figure). The tangent line of the curve describing the arch of the gable is horizontal at the endpoints. The architect wants to describe the curve by the graph of the function $f\,(x)=ax^4+bx^2+c$.

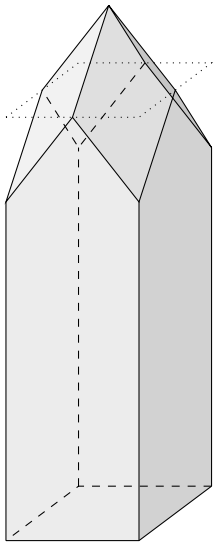


   a) Find the constants a, b, and c in the formula of f.

   b) What is the minimum number of cans of paint needed to paint the gable twice, if 1 m^2 needs 350 cm^3 of paint, and the paint comes in 2-liter, 4-liter, and 5-liter cans?

   (16 points)

9. Prince Architectos and his three sons captured a city with their armies. At three corners of the city wall stood three identical towers in the shape of square prisms; the edges of their bases were 8 m and their heights 40 m. To remember the victory, the three sons rebuilt the three towers, in very similar ways. Along the segments joining the midpoints of neighboring edges of the top face of each original tower, they cut off four identical corners. These were placed on the remaining top face as in the figure, so that the new roof of the tower consisted of rhombi joined to each other.



   The surface area of the tower of the oldest son did not change after the rebuilding. The middle son rebuilt his tower so that its total surface area became as small as possible. On the tower of the youngest son, neighboring roof planes made an angle of $120°$. Find the heights of the rebuilt towers.

   (16 points)

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