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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. Find the exact values of the following expressions without a calculator.

a) $\sqrt{100 \lg(1 - \sin \frac{19\pi}{6})}$ b) $\cos^2 15^\circ \cdot \tan 15^\circ$ c) $\log_\pi \frac{1}{\sqrt{\pi}} + \tan\left(-\frac{11\pi}{4}\right) + 2^{\log_4 9}$

(11 points)

2. Solve the following system of equations over the pairs of real numbers.

$$\begin{aligned}\sqrt{x^2 - 3xy + 4} + x(x - 3y) &= 2 \\ 2x - y &= 3\end{aligned}$$

(12 points)

3. The circle $x^2 + y^2 - 6x - 4y + a = 0$ is given. There is exactly one point on the x -axis from which the tangent lines drawn to the circle make an angle of 60° with each other.

- a) Find the value of the parameter a .
b) For $a = 12$, what angle do the tangent lines drawn to the circle from the origin make with each other?

(14 points)

4. Three positive numbers are three consecutive terms of a geometric sequence. If we add two to the second one, we get three consecutive terms of an arithmetic sequence. If we then increase the first term of this new sequence by 16, we again get three consecutive terms of a new geometric sequence. What are the original three numbers?

(14 points)

Part II

5. Two straight roads cross each other at a right angle at the point A . There is a restaurant on one road 70 km from the crossing, and on the other road 10 km from the crossing. A group starts from each restaurant toward A at the same time. The group from the farther restaurant covers 4 km per hour, and the group from the nearer one 3 km per hour.

- a) After how many hours will they be closest to each other, and what is the smallest distance between them?
b) How many hours after the start will the distance between them be 50 km?

(16 points)

6. We roll a die three times.

- a) What is the probability that the product of the numbers rolled is a prime?
- b) What is the probability that the product of the numbers rolled is a perfect square?

(16 points)

7. We pour water into a container in the shape of a right circular cylinder of radius 10 cm. Then we place into it a right square pyramid all of whose edges are 12 cm. The pyramid sinks to the bottom of the water, its base square stands on the base circle of the cylinder, and the apex of the pyramid sticks out 3 cm above the water. How much does the water level rise?

(16 points)

8. In the equation $\sin^2 x + \sin x + 2^m - 4 = 0$, find the value of the parameter m so that the equation has exactly one root in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

(16 points)

9. Find the area of the finite region bounded by the graphs of the curves $y = x^2 + 4$, $y = -3x + 4$, and $y = x^3$. We rotate this region about the x -axis. What is the volume of the resulting solid of revolution?

(16 points)