

Advanced Level Practice Set - 2010 (March 2010)

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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. The arithmetic sequence (a_n) is given. Prove that the sequence defined by $b_n = a_{n+1}^2 - a_n^2$ is also an arithmetic sequence.

(11 points)

2. Solve the following equation over the real numbers.

$$\frac{\sqrt{x+1}+2}{\sqrt{x+1}-1} = \frac{x+1}{x-2}$$

(12 points)

3. Which are the points $P(x, y)$ for which $||x| - 1| - |y| \leq 0$?

(14 points)

4. An urn contains 7 red and 9 blue balls. We draw five balls one after another, putting the drawn ball back after each draw. What is the probability that there are more red balls than blue ones among the drawn balls?

(14 points)

Part II

5. A telecommunications company has 13 000 subscribers with a monthly fee of 6500 Ft. Market research shows that if the monthly fee were decreased by 100 Ft, the company would get 250 new subscribers, and this would be true for every further decrease by 100 Ft. (The monthly fee at this company is always divisible by 100.) With what monthly fee would the company have the largest revenue, according to the market research?

(16 points)

6. Let A_1, B_1, C_1 be arbitrary points on the sides BC, CA, AB of the triangle ABC . Let $l_a = AA_1, l_b = BB_1, l_c = CC_1$. Prove that

$$\frac{1}{2} < \frac{l_a + l_b + l_c}{a + b + c} < \frac{3}{2}$$

(16 points)

7. We inscribe a hemisphere in a right circular cone so that its circular face lies in the plane of the base circle of the cone, and its spherical surface touches the lateral surface of the cone. The surface area of the cone is to the area of the curved surface of the hemisphere as 18 : 5. What is the vertex angle of the cone?

(16 points)

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8. Bálint and Jonatán play the following game. They roll two dice; if the product or the sum of the numbers rolled is divisible by three, Bálint wins, otherwise Jonatán wins the game. Who has the greater chance of winning?

(16 points)

9. For which real numbers p does the following inequality hold for every real number x ?

$$\frac{2x^2 + 2x + 3}{x^2 + x + 1} \leq p$$

(16 points)