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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. Find the real solutions of the following equations.

a) $(x + 500)^2 = 252\,012 - 1011x$

b) $\lg^2 x + 2011 \cdot \lg x - 2012 = 0$

c) $x + 2011\sqrt{x-7} - 2019 = 0$

(11 points)

2. The first term of a geometric sequence is 2. If we decrease the second term of the sequence by 20, and write the third term in front of the first two, we get three consecutive terms of an arithmetic sequence. Find the original three numbers.

(12 points)

3. In the isosceles triangle ABC , $AB = 6$ and $CF = 9$, where F is the midpoint of the base AB . Starting from the vertex A and going around the triangle in the direction A, B, C , the points dividing the sides into three equal parts are $A_1, A_2, B_1, B_2, C_1, C_2$. Find the area of the quadrilateral formed by the lines C_1B_1, C_2B_2, C_1A_2 , and A_1B_2 .

(14 points)

4. The lower side AB of a rectangle $ABCD$ inclined at 20° is horizontal and 20 meters long. We want to get from the vertex A along a straight line to some point of the side CD . The parallel sides AB and CD are 12 meters apart. Where can we arrive, if the angle between the path and the horizontal may not exceed 12° ?

(14 points)

Part II

5. Consider the circle k with center $K(5, 2)$ and radius $r = 3$. The tangent lines e_1 and e_2 can be drawn from the origin to the circle k .

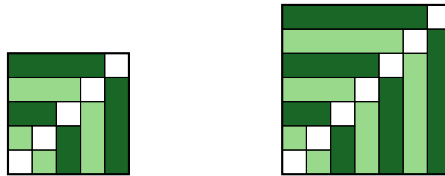
a) Write the equations of e_1 and e_2 .

b) What is the angle between e_1 and e_2 ?

c) Find the area of the plane region from whose points the circle k is seen at an angle of at least 60° and at most 90° .

(16 points)

6. The waiting room of the Tiszai railway station in Miskolc was paved partly with the tiles shown in the figure. On these tiles a 5×5 square grid was colored with three colors (dark green, light green, and white). This pattern can be continued to the right and upward, and the coloring can be made for any odd number greater than five. The figure also shows the 7×7 grid made this way.



- a) What fraction of the square with side 51 units is light green?
- b) What fraction of the square with side 101 units is dark green?
- c) Prove that if the side length of the square increases without bound, then the area of both the light green and the dark green parts approaches half of the area of the square.

(16 points)

7. The graph of the function $f(x) = \frac{x^2}{16} + 1$ defined on $[0, 4]$ is rotated about the x -axis. The surface obtained is the lateral surface of a vessel in the shape of a solid of revolution, whose bottom is the smaller circle. The unit of the coordinate system is 1 dm.

- a) How many liters does the vessel hold, if the thickness of its wall is negligible?
- b) A straight rod of negligible thickness stands at the center of the bottom of the vessel. We tilt the rod so that its lower end does not move. At what height does the rod touch the wall of the vessel?

(16 points)

8. Solve the equation $-x^2 + 12x - 33 = \sqrt{x-3} + \sqrt{x-7}$.

(16 points)

9. We know the lengths of all three sides and the measures of all three angles of a triangle. Show that substituting these values into the formula

$$\frac{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2 + \left(\frac{c}{2}\right)^2}{\cot \alpha + \cot \beta + \cot \gamma}$$

gives the area of the triangle.

(16 points)