

author: Levente Koncz

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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. The common difference of an arithmetic sequence is 56, and its n th term is 2012. The sum of the first n terms of the sequence is 32 000. Find the first term of the sequence.

(12 points)

2. A city park has the shape of an isosceles triangle whose legs are 40 meters long and form an angle of 50° . An arc of radius 35 meters is drawn about the meeting point of the legs. The part inside the arc is sown with grass, and the part outside the arc is planted with flowers.

a) Find the number of flower plants needed for the planting, if they count on 30 plants per square meter.

b) Paths are planned all around the boundary of the park and along the part of the arc inside the park. Find the total length of the paths to be built. (Neglect the width of the paths in the calculations.)

(13 points)

3. How many non-isomorphic simple graphs are there whose degree sequence is

a) 0, 1, 2, 3, 4, 5, 6, 7; b) 2, 2, 3, 3, 3, 4, 5, 5; c) 2, 3, 3, 4, 5, 5, 6;

d) 3, 3, 3, 5, 6, 6, 6; e) 2, 3, 3, 4, 4, 4?

In the cases where no such graph exists, explain why not.

(14 points)

4. We want to survey a rectangular plot of land. For this we choose points A and B , 50 meters apart, in the interior of one side of the rectangle. Let C and D be the vertices of the rectangle on the side opposite to the side containing the two points. We measure the following angles: $\angle CAB = 113^\circ$, $\angle DAB = 48^\circ$, $\angle CBA = 54^\circ$. What is the area of the plot of land?

(12 points)

Part II

5. Danny plays tennis. In a tournament, 63% of his first serves landed in. When his first serve was in, he won 90% of the points. When it was not in, he could serve a second time; 85% of these landed in. When his second serve was in, he won 60% of the points. When neither of his serves landed in, Danny lost the point.

a) What is the probability that Danny lost a point when he was serving?

b) What is the probability that Danny's first serve was in, given that he lost the point?

(16 points)

6. a) Find the value of $\frac{x}{y}$ to three decimal places, if we know that $2^x = 5^y$, where x and y are nonzero real numbers.
b) Solve the following inequality on the largest possible subset of the real numbers.

$$\frac{5}{2} \sqrt{\lg x} + 2 \lg \sqrt{\frac{1}{x}} - 1 < 0$$

(16 points)

7. The region enclosed by the graph of the function $f(x) = 2 \sin x$ and the x -axis on the interval $[0, \pi]$ is divided into two regions by the graph of the function $g(x) = 2 \cos x$. Find the ratio of the areas of the two regions.

(16 points)

8. The diagonals of a cyclic quadrilateral are perpendicular to each other. The intersection point of the diagonals is $E(2, 1)$, and the equations of the sides AB and BC of the quadrilateral are $x + 3y = 10$ and $x - y + 2 = 0$. Find the coordinates of the vertices of the quadrilateral and the equation of its circumcircle.

(16 points)

9. The start of a duathlon race is on a straight sea shore, and the finish is in the water. One way to complete the distance is to first run 4 km along the shore, then turn 90° and swim 1 km out to sea. However, it is allowed to leave the running track at any time and start swimming toward the finish. Our speed is 6 km/h on the sandy shore and 2 km/h in the water. Measured from the start line, at what distance should we head into the water,

- a) if we want to cover equal distances running and swimming;
b) if we want to reach the finish as soon as possible? How many km do we have to swim then, and in how much time can we complete the race this way?

Give the answers to the nearest meter.

(16 points)