Advanced Level Practice Set - 2012 (December 2012)

author: László Számadó
translated from: KoMaL, December 2012, pages 529-531
Pdf file: KoMaL, December 2012, pages 529-531

From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.
The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.
This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. The function $f(x) = \frac{x-671}{x-4} - \frac{x-503}{x-3}$ is defined on the set $\mathbb{R} \setminus \{3, 4\}$.

   a) Find the zeros of the function f.

   b) Is -417 in the range of f? If yes, give the corresponding elements of the domain.

   (11 points)

2. The annual subscription fee of a magazine was 1750 Ft. For the next year it was raised to 1800 Ft.

   a) By what percent was it raised?

   To tell the whole truth: when the magazine was cheaper, it appeared five times a year, with 32, 32, 32, 32, and 28 pages. When it became more expensive, only four issues came out per year, with 36, 48, 40, and 44 pages.

   b) What change did this really mean for the subscribers, per page?

   c) By how much did the number of pages of an issue change on average, comparing the two years?

   d) Give the standard deviation of the numbers of pages of the issues for both years.

   (13 points)

3. Three points are given in the coordinate system:

   $$A(980, -1), \qquad B(981, 1), \qquad C\left(2^{1986}, 2 \cdot 2^{1986}\right)$$

   Reflecting the point C in the midpoint of AB, we get the point D. Find the area of the quadrilateral determined by the points A, B, C, and D.

   (13 points)

4. The lateral surface of a right circular cone, unrolled, is a quarter circle of radius 10 cm.

   a) What is the surface area and the volume of the cone?

   b) The cone is cut into two parts by a plane parallel to the base, at half the height. What is the ratio of the volumes of the two parts?

   (14 points)

Part II

5. a) Prove that

$$a = \sqrt[3]{15 + 4\sqrt{14}} + \sqrt[3]{15 - 4\sqrt{14}}$$

is a root of the equation $x^3 - 3x - 30 = 0$.

b) Prove that the only root of the equation $x^3 - 3x - 18 = 0$ is

$$b = \sqrt[3]{9 + 4\sqrt{5}} + \sqrt[3]{9 - 4\sqrt{5}}$$

(16 points)

6. In her history lesson Ibolya demonstrated indirect presidential elections to her students using the data of an imaginary country.

Bergengocia consists of eight states, so its official name is the United States of Bergengocia (USB for short). In Bergengocia the voters can choose between two parties, and then a given number of electors of the parties elect the president from one candidate of each of the two parties. In each state the number of electors is determined from the data of the last census. The current data are shown in the following table:

State	Voters	Electors
Aliforn	283 821	10
Lorida	234 171	9
Rizon	179 351	7
Olorad	38 021	3
Labama	28 221	3
Regon	26 991	3
Ebrasz	5 261	2
Laszka	4 171	2

The inhabitants of Bergengocia are very conscientious, so everybody exercises the right to vote. Everybody votes for the party whose electors they expect to elect the right president. The party that gets the most votes wins. In each state the winning party gives all the electors, and the electors can then freely choose between the two candidates (they are not bound by party discipline). In each state the presidential candidate supported by the majority of the electors gets the support of all the electors of the state. The candidate supported by the most electors wins the election.

a) On average, how many voters live in each state of USB?

b) In which state does it take the most, and in which the fewest votes to elect one elector?

c) One of the candidates stated that he will surely be the winner, because he is certain of the support of three states. Which three states could he have had in mind?

d) Could someone win this election if fewer than 20% of the voters wanted him to be the president?

(16 points)

7. The price of a coat was originally $20\,000$ Ft, but the store started selling it at a discount. Shortly afterwards, in a new sale, its price became $14\,960$ Ft. In both cases the discount, expressed in percent, was an integer between 10 and 20. Laszlo bought a coat after the first discount. How much could he have paid then?

(16 points)

8. a) The side lengths of a right triangle are consecutive terms of a geometric sequence, and its shortest side is 1 unit long. Compute the lengths of the other two sides of the triangle.

b) The side lengths of a triangle are consecutive terms of a geometric sequence, and its shortest side is 1 unit long. We know that the triangle is not equilateral. Prove that the triangle has no 60° angle.

(16 points)

9. Consider the function f defined on the set of real numbers by $f(x) = x^3 + (p-4)x^2 + p^2x - 2$, where p is a real parameter.

   a) Compute the definite integral $\int_0^4 f(x)\,dx$, if $p = 4$.

   b) Find the value of p for which $x = 2$ is a zero of the function f.

   c) Find the integers p for which the derivative of the function f has no zero.

(16 points)

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