# Advanced Level Practice Set - 2012 (February 2012)

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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.
The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.
This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

## Part I

1. When Johnny crossed the finish line in a cross-country race, the reporter on the spot asked him which place he had finished in. Johnny answered: "If half of those who finished ahead of me had finished behind me, then five times as many would have finished behind me as ahead of me. But if a third of those who finished behind me had finished ahead of me, then 6 more would have finished ahead of me than behind me."

   How many runners took part in the race, and which place did Johnny finish in?

   *(12 points)*

2. A goat is tied to a stake driven into the midpoint of one side of a meadow in the shape of a regular hexagon. The rope is just long enough (when it is taut) for the goat to reach the far endpoints of the two neighboring sides. What percent of the meadow can the goat graze?

   *(12 points)*

3. The universal set is the set of the first $n$ positive integers. Let $A$ be the set of the numbers divisible by 3, $B$ the set of the numbers divisible by 4, and $C$ the set of the numbers divisible by 5.

   a) If $n = 100$, what is the probability that a randomly chosen number is not an element of any of the three sets?

   b) If the set $A \cap B \cap C$ has 2 elements, how many elements can the set $(A \cap B) \setminus C$ have?

   *(13 points)*

4. According to the statistics of the Hungarian State Railways (MÁV), 8% of the passengers ride without a valid ticket.

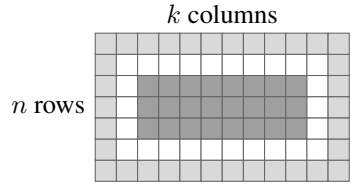
   a) There are 24 passengers in a railway car. What is the probability that the conductor finds a passenger without a ticket in the car?

   b) For how many passengers is the probability that the conductor finds a passenger without a ticket in the car at least 90%?

   *(14 points)*

## Part II

5. A grid of unit squares has $n$ rows and $k$ columns.



The number of squares touching the edge of the grid (light gray in the figure), the number of squares touching those squares (white in the figure), and the number of squares inside the grid (dark gray in the figure) are consecutive terms of an arithmetic sequence. Find the values of $n$ and $k$.

*(16 points)*

6. For which positive integers $n$ is the value of the fraction $\frac{3\sqrt{n}+\sqrt{3}}{4\sqrt{3}-\sqrt{n}}$ a positive integer?

*(16 points)*

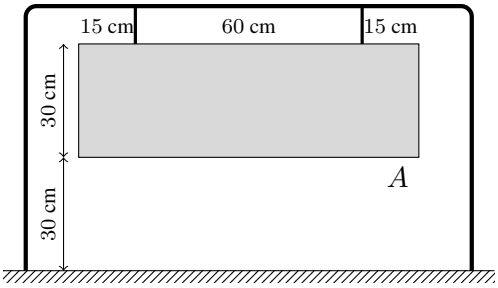
7. The Budapest–Zürich international express train consists of 12 cars: 4 first-class cars, 6 second-class cars, a dining car, and a baggage car.

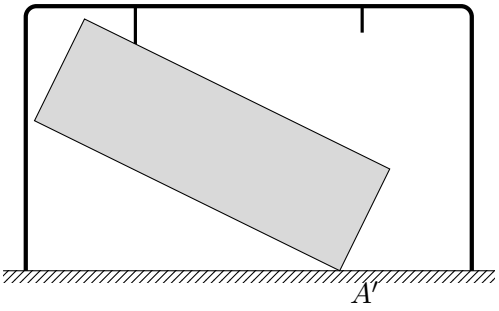
   a) In how many ways can the cars be ordered so that the first-class cars are next to each other, the second-class cars are next to each other, and the dining car is not the last car of the train?

   b) 8 scientists traveled in one of the cars, some of whom already knew each other. One of the scientists (let us call him $A$) knew everyone in the group. There were three scientists among them who knew nobody except $A$, while the other four knew each other. Those who did not know each other introduced themselves with a handshake. How many handshakes were there?

*(16 points)*

8. In front of the entrance of a restaurant, vertical rods were welded to a metal frame standing on level ground, and the daily menu was posted on a board screwed to the rods. After a while one of the bearings broke (see the figure), and so the board tipped down to the ground.




   a) Find the position $A'$ of the vertex $A$ of the board on the ground after it tipped.

   b) Through how many degrees did the board turn as it tipped?

*(16 points)*

9. A semicircle is drawn about the midpoint of the base of an isosceles triangle, tangent to the legs of the triangle. Find the angles of the triangle if the area of the semicircle is the largest possible percent of the area of the triangle.

*(16 points)*

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