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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. Solve the following equation over the real numbers.

$$x^2 + \frac{2x}{x+5} + \frac{1}{x^2 + 10x + 25} = 25$$

(12 points)

2. The parallel sides of a trapezoid are a and c . How long is the segment parallel to the bases that bisects the area of the trapezoid?

(12 points)

3. Solve the equation $\log_x(5x^2) \cdot \log_5^2 x = 1$ over the real numbers.

(13 points)

4. A snack bar sells sandwiches and pizzas. The owner makes a 30% profit on a sandwich and a 50% profit on a pizza. One day the owner sold as many sandwiches as pizzas, and so made a 34% profit. The next day, however, customers bought twice as many pizzas as sandwiches.

- a) What was the owner's profit on the goods sold on the second day?
- b) What would the profit have been if the owner had sold twice as many sandwiches as pizzas?

(14 points)

Part II

5. A billiard ball is at the point P of a circular billiard table of diameter 4 m, 0.5 meters from its center O . The ball must be shot so that after two bounces it passes through the point P again. What is the angle between the direction of the shot and the direction PO ?

(16 points)

6. The general term of a sequence is given by the formula

$$a_n = \frac{2n^2 + 3n - 1}{n^2 + 4}$$

- a) Find the limit $\lim_{n \rightarrow \infty} a_n$.

- b) Find the threshold index N from which on the terms of the sequence differ from the limit of the sequence by less than 10^{-2} .

(16 points)

7. The school cafeteria usually orders potatoes from two places in 10 kg bags. Twice as many are ordered from supplier A as from supplier B. Supplier A delivers the order within a week with probability 80%, and supplier B with probability 60%.
- a) What is the probability that for a randomly chosen 10 kg bag we must wait more than a week?
 - b) If we know that the potatoes were delivered within a week, what is the probability that they were ordered from A?
 - c) If in case of a late delivery the potatoes are sold at a 10% discount, and a 10 kg bag costs 800 Ft at both suppliers, then what is the expected price paid for a 10 kg bag?

(16 points)

8. The vertex of a parabola with axis parallel to the y -axis is $T(1, 4)$, and the slope of the tangent line at the point of the parabola with x -coordinate 2 is 6. Find the area of the plane region bounded by the y -axis, the parabola, and the tangent line at the point of the parabola with x -coordinate 2.

(16 points)

9. Find the equation of the line through the point $P(1, 2)$ whose segment between the lines $x - y + 5 = 0$ and $x - y = 2$ is 5 units long.

(16 points)