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translated from: KoMaL, March 2012, pages 132-133

Pdf file: KoMaL, March 2012, pages 132-133

From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. Pharmacies sell a cold-relief powder, to be dissolved in hot water, in identical small sachets, packed six or ten to a cardboard box. A box of six costs 1130 Ft, and a box of ten costs 1640 Ft. Since the material cost of the boxes is minimal, assume that their price is the same regardless of their size, and that the price of the sachets of powder is also the same. What would a box of fourteen cost then?

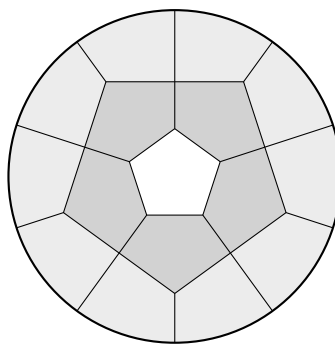
(11 points)

2. Solve the following equation.

$$\lg(1-x) \cdot \sqrt{x^2 - 3x - 28} \cdot \cos\left(x + \frac{\pi}{6}\right) = 0$$

(13 points)

3. Bettina bought the latest mosaic powder. The figure shows a sketch of the pattern of the cylindrical compact of radius 4 cm, seen from above.



- a) The pentagon in the middle of the pattern is regular, and together with the pentagons attached to it, it forms a larger regular pentagon. What would be the ratio of the sides of these two regular pentagons if the 16 regions of the compact had equal areas?

- b) Under the previous conditions, find the side length of the small regular pentagon in the middle to the nearest millimeter.

(13 points)

4. Solve the equation

$$2 \sin^2 2x - 2 \cos^2 x = 1$$

(14 points)

Part II

5. Tangent lines are drawn to the parabola $y = x^2 - 4x + 8$ at its points with x -coordinates 0 and 4.
- What is the angle between the two tangent lines?
 - What is the area of the region bounded by the two tangent lines and the parabola?
- (16 points)*
6. A pump can lift 4.8 m^3 of water per hour from a well. The water is let into a cylindrical pool whose top is a circular slab of area 154 m^2 . This slab also covers the 12 cm thick side wall of the pool.
- By how many centimeters does the water rise in the pool in 2.5 hours?
 - The amount of water pumped in 15 minutes is filled into 25 liter and 60 liter containers. How many containers of each kind could there have been, if they were exactly filled with this amount of water?
- (16 points)*
7. On two neighboring sides of a rectangular lot, we mark a point 10 meters and a point 24 meters from one of the vertices. We want to build a pool in the right triangle cut off by the line connecting the two points. According to one plan, we would build the largest circular pool; according to the other plan, we would build the largest rectangular pool with one full side on the hypotenuse of the right triangle. In which case, and by how much, would the water surface be larger?
- (16 points)*
8. The coordinates of three vertices of a hexagon are, in some order, $(-1, -1)$, $(2, 8)$, $(7, 3)$. Find the coordinates of the missing vertices if we know that each of the unknown points forms an isosceles trapezoid with the three given vertices.
- (16 points)*
9. Let
- $$A = \frac{6n^2 - 15n - 9}{4n^2 - 18n - 10}$$
- What is the limit $\lim_{n \rightarrow \infty} A$?
 - Find the integers n for which A is also an integer.
 - Solve the equation $A = 3$.
- (16 points)*