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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. Which three natural numbers are consecutive terms of an arithmetic sequence, if the sum of their squares is 264, and their product equals the quotient of 1792 and the middle number?

(10 points)

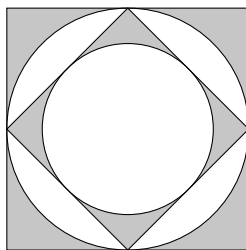
2. What is the area of the tangential trapezoid one of whose legs forms an angle of 120° with the 7 cm long parallel side, and whose inscribed circle has radius 5 cm?

(13 points)

3. The figure shows a square with side 1 unit, its inscribed circle, the square determined by the midpoints of its sides, and the inscribed circle of that square too.

a) What percent of the large square was colored gray?

b) We repeat this procedure infinitely many times. What percent of the large square is colored gray this way?



(14 points)

4. Find the two consecutive integers between which the following sum lies.

$$\lg \left(11 \cdot \sqrt{\frac{1}{3}} \right) + \lg \left(11^2 \cdot \sqrt{\frac{3}{5}} \right) + \lg \left(11^3 \cdot \sqrt{\frac{5}{7}} \right) + \cdots + \lg \left(11^{500} \cdot \sqrt{\frac{999}{1001}} \right)$$

(14 points)

Part II

5. Spherical chocolates of radius 2 cm are packed five at a time into boxes shaped like square-based pyramids, so that four are at the bottom and one on top. Each lower sphere touches the upper one and two neighboring lower ones, and also touches the base of the pyramid and two neighboring lateral faces. The upper sphere touches all four lateral faces.

a) How tall is the box?

b) What is the surface area of the box?

(16 points)

6. Solve the following equations over the real numbers.

a) $\sqrt{x} + \sqrt{2x+1} = \sqrt{3x+13}$

b) $\frac{\log_2(x^3 + 26)}{\log_4(x+2)} = 6$

c) $2\sin^2 x + \sin^2 2x = 2$

(16 points)

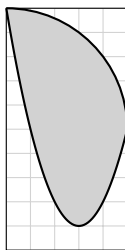
7. In the coordinate system the circles $k_1: x^2 + y^2 = 9$ and $k_2: (x-17)^2 + (y-7)^2 = 100$ are given. Prove that the segment joining the centers of the two circles subtends a right angle at the intersection of the horizontal common external tangent and the common internal tangent with positive slope of the two circles.

(16 points)

8. A tailor uses the pattern shown in the figure for a carnival butterfly badge, and cuts it out of a rectangular piece of fabric bought in the store. The pattern consists of an arc of a parabola and an arc of a circle.

a) What percent of the fabric will be waste?

b) Find all the intersection points of the parabola and the circle.



(16 points)

9. At the 2012 London Olympics the Hungarian team won 8 gold, 4 silver, and 5 bronze medals. There was no event in which two Hungarian medals were won.

a) In how many ways can we list the events in which our team won a medal, so that first the events of the gold medals, then those of the silver medals, and finally those of the bronze medals are named?

b) In how many ways can we list the above 17 events, if among the events of medals of the same color we always list first the one that ended earlier?

c) What is the probability that choosing three of our medal-winning events at random, the corresponding medals will be of different colors?

(16 points)