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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. In the school chess championship everybody played exactly once against everybody else. When 42 games had been played, everybody still had four games left. How many players took part in this championship?

(11 points)

2. Graph the following functions.

a) $f(x) = |x + 2| + |x - 1|$ on the interval $[-2, 1]$;

b) $g(x) = (\tan x + \cot x) \sin x \cos x$ on the interval $[-\pi, \pi]$;

c) $h(x) = \log_{2013} x \cdot \log_x 2014 \cdot \log_{2014} 2013$ on the interval $]0, 3]$.

(13 points)

3. Solve the following equation, where n is an arbitrary positive integer greater than 1.

$$\sqrt{x^2 + x - 2} + \sqrt{x^2 + 2x - 3} + \cdots + \sqrt{x^2 + nx - (n + 1)} = 0$$

(13 points)

4. Write the equation of the circle that passes through the point $A(-7, 5)$ and touches the circle $(x - 2)^2 + (y - 3)^2 = 17$ at the point $E(-2, 2)$.

(14 points)

Part II

5. The base AB of the isosceles triangle ABC is 26 cm, and its legs are 85 cm long. Let F be the midpoint of the base AB , K the center of the inscribed circle, O the center of the circumscribed circle, S the centroid, and M the orthocenter.

a) What is the perimeter of the triangle ASO ?

b) How long is the segment FK ?

c) How long is the segment MF , and at what angle is the leg AC seen from the point M ?

(16 points)

6. Solve the following system of equations over the set of pairs of integers.

$$\begin{aligned} 3^{3x-1} &= 2y^3 - 11y - 693 \\ x &= \log_3(y + 1) \end{aligned}$$

(16 points)

7. a) Find the integral of the function $f(x) = x^2 - 10x + 27$ on the interval $[3, 6]$.
b) By how much must the given interval be shifted so that the integral is 18?

(16 points)

8. The following data set is given: 1, 1, 2, 4, 8, 8, 8, 9, 13.

- a) Prove that for this data set, the sum of the distances from the mean of the numbers smaller than the mean equals the sum of the distances from the mean of the numbers greater than it.
b) Find the average of the data set for which the sum of the absolute distances of the data from it is minimal.
c) Find the average of the data set for which the sum of the squares of the distances of the data from it is minimal.

(16 points)

9. Find the acute angle x for which the expression

$$\frac{4 \sin^2 x - 4 \sin^2 x \cos^2 x + 1}{\sin^2 x}$$

is minimal. What is the smallest value?

(16 points)