Advanced Level Practice Set - 2013 (December 2013)

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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.
The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.
This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. In a television talent show, the singers of four mentors compete. The ranking is decided by the phone votes of the viewers. To make it more exciting, the host gives each mentor a little information about the state of the vote. These statements were made, in this order:

   Of your three contestants, exactly two are not in one of the last three places.

   Of your two contestants, exactly one is in one of the last three places.

   Of your two contestants, at least one is in one of the last three places.

   Of your two contestants, at least one is certainly not in one of the last three places.

   a) Can any of the four mentors think now that none of his or her contestants is in one of the last three places?

   b) How many rankings are possible, based on the statements made?

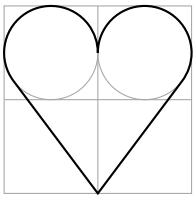
   (11 points)

2. Solve the following equation. (lg denotes the logarithm in base 10.)

$$\lg(2-x)\cdot\sqrt{6-x-x^2}\cdot\sin\left(x-\frac{\pi}{6}\right)=0$$

   (12 points)

3. There is a heart shape on Enikő's T-shirt. The heart can be drawn into a square with 24 cm sides, as shown in the figure. What are the perimeter and the area of this heart?



   (14 points)

4. We chose eight cities such that there is a direct flight between any two of them. On a winter day, because of extreme weather, flights are suspended on nine of all these routes. What is the probability that of four randomly chosen routes, there are flights on exactly two?

   (14 points)

Part II

5. Solve the following equation over the set of real numbers.

$$\left(\frac{\sqrt{5}}{4}\right)^{\frac{12-2\lg x}{-5+\lg x}-\frac{4}{1+\lg x}} = 10.24$$

(16 points)

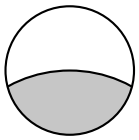
6. We cut the cube $ABCDEFGH$ into two pieces with the plane through the vertex A, the midpoint P of the edge BF, and the center Q of the face $ADHE$.

   a) Find the area of the cross-section in terms of the edge of the cube.

   b) What is the ratio of the volumes of the two pieces?

(16 points)

7. The figure shows a sketch of one of Ibolya's earrings. The circular arc that passes through the center of the circular disk has twice the radius of the disk. What percent of the earring is darker?



(16 points)

8. About a cubic function f we know that $f(2) = f(4) = 4$, the coefficient of its cubic term is 1, and

$$\int_2^6 f(x)\,dx = 16$$

   Find the equation of the tangent line to the graph of f at the point with x-coordinate 5.

(16 points)

9. Solve the following equation.

$$\left(x^2 - 2013^2\right)^2 - 1 = 8052x$$

(16 points)

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