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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. How many different cuboids are there whose edge lengths are integers, whose volume is 15 000 units, and for which the greatest common divisor of the lengths of the three edges is 5?

(12 points)

2. Solve the following equations over the real numbers.

a) $\sqrt{x - \sqrt{x + 2}} = 2$

b) $\log_2 x + \log_4 x + \log_8 x = \log_{16} x^3$

(13 points)

3. Observing the habits of our family, the probability that we go for a walk on a summer afternoon is 0.6. If we stay at home, the probability that we eat ice cream is 0.25. The probability that we eat ice cream is 0.2. What is the probability that on a summer afternoon

- a) we stay at home and eat ice cream;
- b) we go for a walk and eat ice cream;
- c) we eat ice cream, given that we go for a walk?

(13 points)

4. The function $f: x \mapsto x^3$ is given.

- a) Give the equations of the tangent lines to the graph of f whose slope is 12.
- b) Compute the distance between the points of tangency.
- c) Find the distance between the tangent lines.

(13 points)

Part II

5. A ball was put into a vessel whose inside has the shape of a frustum of a cone, and which was filled with water up to a height of 3 cm. After the ball was put in, the water just did not overflow. The radii of the bottom and top circles of the frustum are 2 cm and 5 cm, and its height is 4 cm. What is the radius of the ball, if it is immersed in the water up to $\frac{2}{3}$ of its height?

(16 points)

6. In the new building of the Újpest Technical School of the Wood and Metal Industry, teaching began in 1927. The school was built on a pentagonal lot, about which the local history newspaper writes: “Görgey Road and Szent Imre Street form an angle of 49.4° with each other, with straight lot lines of 229 m and 195 m. The lot line on Corvin Street breaks toward Görgey Road at an angle of 44.1° after 104 m, then takes a northern direction at a reflex angle for 49 m, and from there closes in a short section to the endpoint of the lot line on Szent Imre Street.” We also know that Görgey Road forms an angle of $+2.9^\circ$ with the east–west direction, and that the southwest corner of the lot is the intersection of Görgey Road and Corvin Street. Compute the area of the lot.

(16 points)

7. Consider the complete graph on 6 vertices.

- How many cycles with 3, 4, 5, and 6 vertices are there in this graph?
- How many cycles with 3, 4, 5, and 6 vertices will there be, if we delete one edge?

(16 points)

8. About each vertex of a regular n -gon with sides of length 2 ($n \geq 3$), as center, we draw a circle of unit radius.

- Give, as a function of n , the radius r_n of the circle that touches each of the circles drawn about the vertices of the polygon, inside the polygon.
- For which values of n will r_n be greater than 2013?

(16 points)

9. A university student in Bergengocia took out a student loan during the 3 years of studies. From September of the first academic year, the student received 25 000 Ft on the first day of every month. Interest is computed monthly by proportional (linear) interest, that is, the bank computes the monthly interest by the formula $\frac{t \cdot k}{12}$, where t is the principal owed and k is the current annual interest rate. Interest not yet paid in a given year is capitalized every year on December 31. Suppose that the interest rate is 9% during the whole time the loan is taken out and repaid.

- What is the principal owed by the graduate on the first day of the month (July) after finishing the studies?
- From September 1 after finishing the studies until the debt is fully paid, the graduate repays 25 000 Ft each month (or the remaining amount in the last month). The amount repaid each month first reduces only the accumulated interest, and after that is used up, it reduces the principal each month as well. How many months does it take to repay the whole loan this way?

(16 points)