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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.

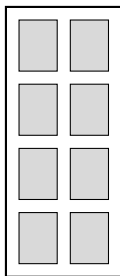
This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. In the triangle ABC inscribed in a circle of radius r , $AB = r$ and $AC = r\sqrt{3}$. How long is the side BC ?

(11 points)

2. The figure shows a sketch of a paneled door 90 cm wide and 210 cm tall.



The eight identical rectangular panels take up exactly half of the door leaf. The strips between and beside the panels have the same width everywhere. What is this width?

(13 points)

3. Solve the following equation.

$$(x - 5 - 2\sqrt{x - 2})(x - 10 - 4\sqrt{x - 5}) = 0$$

(13 points)

4. The function $f:]-3, 5[\rightarrow \mathbb{R}$, $f(x) = \left| \left| (x - 1)^2 - 1 \right| - 3 \right| - 5$ is given.

- Find the zeros of the function.
- Give the coordinates of the lattice points that lie on the graph of the function.
- On which intervals is the function strictly decreasing?

(14 points)

Part II

5. About the positive digits a , b , and c we know that $a + b + c = \overline{ab}$ and $a^2 + b^2 + c^2 = \overline{b^2a}$ (where $\overline{ab}$ and $\overline{b^2a}$ are two-digit numbers). Find these digits.

(16 points)

6. The regular tetrahedron $ABCD$ is cut by a plane. The cutting plane cuts the three edges starting from one vertex in the ratios $1 : 1$, $2 : 1$, and $3 : 1$, measured from the common vertex. Find the ratio of the surface areas of the tetrahedron cut off and the original tetrahedron.

(16 points)

7. Solve the equation

$$\sin^4 x + \sin^4 \left(x + \frac{\pi}{4} \right) + \sin^4 \left(x - \frac{\pi}{4} \right) = \frac{3}{2}$$

over the real numbers.

(16 points)

8. Consider the parabolas $y = (p - 1)x^2 + 2x - (p + 1)$, where p is a real parameter and $p \neq 1$.

- Which of these parabolas cut the x -axis at two points with integer coordinates, or touch it at a point with integer coordinates?
- Write the equation of the tangent line to the parabola with $p = 2$ at its point with first coordinate 4.
- Find the area under the parabola with $p = 3$ on the interval $[1, 3]$.

(16 points)

9. For three different right circular cones we know that the radii of their base circles, and also the slant heights of the cones, are three consecutive terms of arithmetic sequences with the same common difference. Show that the surface areas of the cones cannot be three consecutive terms of an arithmetic sequence.

(16 points)