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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. We wrote the differences of neighboring terms of a geometric sequence (with positive terms) one after the other. Prove that the sequence obtained this way is also a geometric sequence, if $q \neq 1$. Give the first term and the common ratio of the geometric sequence obtained this way.

(11 points)

2. a) Réka sells environmentally friendly laundry gel in 3 liter packages. She put forty-five colored slips into a small basket. Of these slips, eighteen say “one gift”, fifteen “two gifts”, and twelve “three gifts”. Before buying, customers draw a slip, and according to the text on it they get gifts in the following order: a 810 Ft all-purpose cleaner, a 510 Ft fabric softener, a 290 Ft dishwashing liquid. Then they put the slip back into the basket.

Before buying, Laszlo also draws a slip. What is the probability that he will get three gifts?

What should the price of one liter of the laundry gel be, if the distributor wants an average revenue of 535 Ft per liter?

- b) We forgot the four-digit code needed to turn on the phone. We only know that it consisted of four consecutive positive digits, but the digits follow each other in neither increasing nor decreasing order. How many such four-digit numbers are there?

(13 points)

3. At each vertex of a regular hexagon $ABCDEF$ with side length 6 units, lying in a horizontal plane, a segment stands perpendicular to the plane of the hexagon, all on the same side of the plane. The lengths of these segments are

$$AA' = 2, \quad BB' = \frac{9}{2}, \quad CC' = 7, \quad DD' = \frac{19}{2}, \quad EE' = 12, \quad FF' = \frac{29}{2}$$

- a) How long is the closed broken line $A'B'C'D'E'F'$?

- b) Find the pairs of points of $A'D'$, $B'E'$, $C'F'$ that lie exactly above each other, and find the distances between these pairs of points.

(13 points)

4. Consider the sequence given by its general term

$$a_n = a \cdot n^2 + (2a + b) \cdot n - (b^2 - b - a)$$

Show that if a and b are integers, then the sum of any five consecutive terms of the sequence is divisible by 5.

(14 points)

Part II

5. Solve the following equations.

a) $2^{3x} - 2^{2x} - 2^{x+1} = 0$

b) $\sin 3x - \sin 2x - \sin x = 0$

(16 points)

6. The longer edge of a board 0.5 meters wide and 1.2 meters long rests on the horizontal floor, and the edge parallel to it is at a height of 30 cm.

a) What is the angle of elevation of an ant going up along a diagonal of the board?

b) How does this angle change, if the ant goes from one of the points dividing the lower edge into three equal parts to the non-opposite point dividing the upper edge into three equal parts?

c) Find the angle between the two paths.

(16 points)

7. Solve the following equation.

$$x - 8 - \frac{560}{x - 32} = 19 \cdot \sqrt{\frac{x - 8}{x - 32}}$$

(16 points)

8. For the triangle with vertices $A(1, 2)$, $B(12, 4)$, and $C(4, 8)$, find

a) its centroid;

b) its largest angle;

c) the value of $CP^2 + CQ^2$, where P is the intersection of the interior angle bisector from the vertex C with the line AB , and Q is the intersection of the exterior angle bisector from the vertex C with the line AB .

(16 points)

9. Find the values of the following definite integrals.

a) $\int_{-1}^4 (x^2 - 2x + 2) dx$

b) $\int_{-\pi}^{\pi} (x + \cos x) dx$

c) $\int_1^2 \left(e^x - \frac{1}{x} \right) dx$

d) $\int_0^1 (2x + 1)^4 dx$

(16 points)