Advanced Level Practice Set - 2013 (November 2013)

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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.
The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.
This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. Of two convex polygons, the first one has 540 more diagonals than the second one, and the first one has three times as many vertices as the second one. How many vertices does each polygon have?

(11 points)

2. The following notice can be read on the door of the snack bar at the Miskolc railway station:

Open 3:30 – 23:30. Closed between 7:30 – 8:30 and 19:30 – 20:30 because of the change of shifts!

a) What is the probability that we find the snack bar open, if we arrive at its door at a random time between 7 in the morning and 9 in the evening?

b) We bought a pastry for 200 forints. We paid with the exact amount. In how many ways could we do this, if we did not use any coin smaller than 20 forints, and the order does not matter? (Hungarian coins are worth 5, 10, 20, 50, 100, and 200 forints.)

c) The next customer wants to buy three pastries. The choices are: walnut crescent, jam-filled pastry, poppy seed roll, sour cherry strudel, and cocoa roll. What is the probability that we guess correctly what he will buy, if we assume that each of his possible choices is equally likely?

(12 points)

3. Four points are given in the coordinate plane: $A_1\,(0,5)$, $B_1\,(1,7)$, $A_2\,(3,4)$, $B_2\,(5,5)$. We know that A_1A_2 and B_1B_2 are chords of two concentric circles.

a) Where is the common center of the circles?

b) We reflect the polygonal path $A_1B_1A_2B_2$ across the bisector of the first quadrant (the line $y = x$), and then we reflect both the original and the reflected figure across both axes and across the origin. What are the perimeter and the area of the polygon we get this way?

(14 points)

4. In the sequence 1, 6, 13, 24, 37, 54, 73, 96, 121, ..., each term after the second one is obtained by adding six times the index of the term between them to the term two places before it. We already know that the terms in even positions can be written as a quadratic function of their index. What is the 100th term of the sequence?

(14 points)

Part II

5. Three lines are given by their equations:

$$a:\ x+3y=7, \qquad b:\ 3x-y=-7, \qquad c:\ 3x+4y=8$$

a) Show that the three lines determine a right triangle.

b) How far is the vertex of the right angle from the midpoint of the opposite side?

c) What is the radius of the inscribed circle of the triangle?

(16 points)

6. A function is given by the formula

$$f(x)=\frac{\dfrac{x+2}{x-2}-\dfrac{x-2}{x+2}}{\dfrac{x-2}{x+2}+1}$$

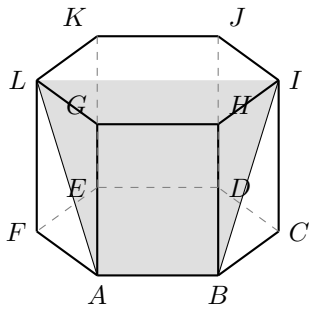
a) Find the largest set that can be the domain of f.

b) Find the range of f.

c) How many lattice points (points with integer coordinates) lie on the graph of f? List these points.

(16 points)

7. László is designing a box for a game called "Exact Step"; the box is a right prism whose bases are regular hexagons. The bottom face of the box is the regular hexagon $ABCDEF$, and the top face is the regular hexagon $GHIJKL$ (with G above A, H above B, and so on).



a) What would be the area of the side face $ABHG$, if the area of the cross-section $ABIL$ is 72 cm^2 and the distance between the parallel lines AB and IL is 8 cm?

b) What would be the angle between the base and the plane $ABIL$, if $AL=\sqrt{2}\cdot AB$?

(16 points)

8. a) The tangent lines to the graph of the function $f(x)=4-x^2$ at its points with x-coordinates 1 and -1, together with the x-axis, determine a triangle. We cut the triangle along the graph of the function. What is the total area of the regions on the right, on the left, and on the top of the curve?

b) The cross-section of a tunnel can be well approximated by the region between the graph of $f(x)=4-x^2$ and the x-axis, where the unit of the coordinate system is 1 meter. What is the largest possible volume of a 6 meter long rectangular box that can be pushed through the tunnel, sliding on one of its faces?

(16 points)

9. A 5 meter tall vertical pole stands on level ground. Its top is seen from the point A of the ground at an angle of elevation of 6.5°, and from the point B at an angle of elevation of 5.6°. From the top of the pole, the segment AB is seen at an angle of 80°.

a) Describe briefly how we would find the area of the triangle ABC (C is the foot of the pole), if we knew the length of AB.

b) How long is the distance AB?

(16 points)

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