Advanced Level Practice Set - 2013 (October 2013)

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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.
The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.
This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. In one month, Péter made 18 calls and talked for 60 minutes within his phone network. Outside the network, he made 11 calls and talked for 45 minutes. He sent 10 text messages. Compare the two calling plans below ("Haver 1" and "Haver 2"; *haver* means *buddy*), and show by computation which one would have been cheaper for him in that month.

Name of the plan	"Haver 1"	"Haver 2"
Monthly fee, usable for calls	1710 Ft 100% within the network	3850 Ft 60% within the network 40% outside the network
Rate per minute until the monthly fee is used up	38 Ft	35 Ft
Rate per minute after the monthly fee is used up	40 Ft	38 Ft
Text message (SMS)	40 Ft	38 Ft
Connection fee per call	3 Ft	3 Ft

The monthly fee can not be used for the connection fees; those are always billed on top of the monthly fee.

(11 points)

2. A box has the shape of a prism with a square base, and the edge of its base is 8 cm. Two coins of equal diameter fit exactly on the square bottom of the box, lying flat, so that each coin touches two neighboring side faces of the box, and the two coins touch each other. What is the diameter of the coins?

(12 points)

3. Solve the following equations over the set of real numbers.

a) $\cos 2x + 5\sin x + 2 = 0$ *(7 points)*

b) $2\log_5 x - \dfrac{3}{\log_5 x} = 5$ *(7 points)*

4. Thirty students wrote a 10-point test. The results were the following:

score	3	4	5	6	7	8	9	10
frequency	2	3	7	2	2	5	5	4

a) The teacher congratulated the class, because $\frac{3}{5}$ of the students scored more than one of the averages (mean, median, or mode). Which one did the teacher mean? *(6 points)*

b) Two students wrote the test later. With their scores included, the median increased by 0.5 and the mean increased by 0.075. What could their scores be? *(8 points)*

Part II

5. In a card game, the cards are numbered from 1 to 104, and each card carries a number of penalty points according to the following rules:
   - card number 55 carries 7 penalty points;
   - the other cards whose number ends in 5 carry 2 penalty points each;
   - the other cards with a two-digit number made of two equal digits carry 5 penalty points each;
   - the cards whose number ends in 0 carry 3 penalty points each;
   - all other cards carry 1 penalty point each.

   At the beginning of the game, 4 cards are turned face up, every player gets 10 cards, and the remaining cards go to the stock.

   a) How many penalty points are on the cards altogether? *(4 points)*

   b) What is the probability that each of the four cards turned face up carries more than one penalty point? *(6 points)*

   c) With six players, what is the probability that there is at least one card in the stock that carries 5 or more penalty points? *(6 points)*

6. A factory makes one-liter measuring cups in the shape of a cylinder. What should the radius of the base and the height of the cylinder be, so that the amount of material used is minimal,

   a) for a cup without a lid; *(8 points)*

   b) for a cup with a lid? *(8 points)*

   Give the answers to the nearest millimeter.

7. The parabola with equation $y = ax^2 + bx + c$ passes through the point $(8, 0)$ and through the origin. The slope of its tangent line at the origin is 2.

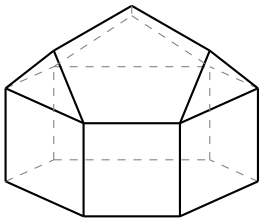
   a) Find a, b, and c. *(6 points)*

   b) We draw a secant line through the origin and the point of the parabola with x-coordinate 6. What is the area of the region enclosed by this secant line and the parabola? *(10 points)*

8. a) Prove that $64 \mid 9^n - 8n - 1$ for every natural number n. *(8 points)*

   b) Is it true that $2013^2 + 2014 \mid 2013^{2014} - 2013$? Justify your answer. *(8 points)*

9. At the top vertex of a structure that serves as a shelter for hikers, three regular pentagons meet, as shown in the figure. The sides of the regular pentagons are 1.5 m long. The side faces of the structure are rectangles, whose sides perpendicular to the ground are also 1.5 m long.



   a) How tall is the structure? *(8 points)*

   b) What is the area of the base of the structure? *(8 points)*

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