Advanced Level Practice Set - 2025 (November 2025)

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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.
The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.
This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. The arithmetic mean of two positive numbers is 205, and the difference of their arithmetic and geometric means is 160. Find the two numbers.

(11 points)

2. a) Find the real number x, if $\overrightarrow{AB} \cdot \overrightarrow{AC} = 0$, where $A\,(x, 7)$, $B\,(4, -1)$, and $C\,(x - 11, -4)$. *(8 points)*

   b) Find the centroid of the triangle ABC obtained in part a). *(2 points)*

   c) Find the area of the triangle ABC obtained in part a). *(3 points)*

3. a) Find the range of the function $f : \mathbb{R} \to \mathbb{R}$, $f\,(x) = -3\cos(4x) + 2$. *(6 points)*

   b) Show by computation that the function $g : \mathbb{R} \to \mathbb{R}$, $g\,(x) = 2025x^3 - 2026x$ is odd. *(6 points)*

4. a) A rectangular plot of land lies along a straight river. The side parallel to the river is four times as long as the other side. There is no fence on the river side, and the total length of the fence along the other three sides is 600 meters. Find the area of the plot. *(3 points)*

   b) Another rectangular plot by the river is also fenced with 600 meters of fence, with no fence on the river side, and so that the area of the plot is as large as possible. Find the area of this plot. *(8 points)*

   c) How many divisors does $-45\,000$ have? *(4 points)*

Part II

5. a) Consider the line $e : x - y = 8$ and the circle $k : x^2 + y^2 - 6x + 4y + 4 = 0$. Determine the relative position of the line e and the circle k, and find their common points, if there are any. *(8 points)*

   b) Consider the points $P\,(3, -5)$ and $Q\,(6, -2)$ in the coordinate plane. Let P_1 and Q_1 be the perpendicular projections of P and Q onto the x-axis, and P_2 and Q_2 their perpendicular projections onto the y-axis. Find the perimeter of the quadrilateral $Q_1P_1Q_2P_2$. *(5 points)*

   c) Find the area of the quadrilateral $Q_1P_1Q_2P_2$. *(3 points)*

6. a) Anna, Bogi, and Cili are cousins. In six years Bogi will be as old as Anna is now. In seven years Anna will be 9 times as old as Cili is now. In eight years the three of them together will be 42 years old. How old are the girls now? *(10 points)*

   b) Solve the inequality $\left(\frac{2}{5}\right)^{\ln x} \geq 6.25$ over the set of real numbers. *(6 points)*

7. a) Prove that the decimal number $2.\dot{0}2\dot{5} = 2.025025025\ldots$ is rational. *(6 points)*

   b) Show that $2\lg 7$ is irrational. ($\lg$ denotes the base 10 logarithm.) *(5 points)*

   c) Compute the indefinite integral $\int \left(5x^2 - 3x^9 + 8x\right) dx$. *(5 points)*

8. Consider the functions $f(x) = x^3$ and $g(x) = 2x + 1$, defined on the set of real numbers. Give the rules and the derivatives of the following composite functions.

   a) $(f \circ g)(x)$ *(2 points)*

   b) $(f \circ g)'(x)$ *(3 points)*

   c) $(f \circ f)(x)$ *(2 points)*

   d) $(f \circ f)'(x)$ *(2 points)*

   e) $(g \circ f)(x)$ *(2 points)*

   f) $(g \circ f)'(x)$ *(2 points)*

   g) $(g \circ g)(x)$ *(2 points)*

   h) $(g \circ g)'(x)$ *(1 points)*

9. a) Töhötöm drew the interval $[-5, 8]$ on the number line, and with his eyes closed he pointed at one of its points. What is the probability that he pointed at a point of the interval $[-1, 2]$, if the probability is proportional to the length of the interval? *(3 points)*

   b) Prove that the intervals $[-5, 8]$ and $[-1, 2]$ have the same cardinality. *(5 points)*

   c) In how many ways can we choose exactly two numbers from the first five rows of Pascal's triangle so that their sum is even? (Two choices are different if at least one of the numbers is taken from a different place in Pascal's triangle.) *(4 points)*

   d) In how many ways can we choose at least seven numbers from the first five rows of Pascal's triangle so that their product is odd? (Two choices are different if at least one of the numbers is taken from a different place in Pascal's triangle.) *(4 points)*

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