Advanced Level Practice Set - 2025 (October 2025)

author: László Németh (Fonyód)
translated from: KoMaL, October 2025 (issue 2025/7), on the KoMaL website
Solutions: KoMaL website

From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.
The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.
This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. Solve the following equations over the set of real numbers.

   a) $\frac{x}{x-1}+\frac{2x+1}{x+1}=\frac{3x+5}{x^2-1}$ *(5 points)*

   b) $\cos 2x+2\sin x+3=0$ *(5 points)*

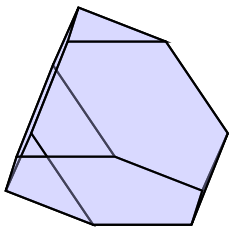
2. Consider the functions $f(x)=|x+1|$ ($x\in\mathbb{R}$) and $g(x)=\sqrt{x}-2$ ($x\in\mathbb{R}$, $x\geq 0$).

   a) Find the domain and the rule of the inverse of the function g. *(4 points)*

   b) Graph the function $f\circ g$ on the interval $[0,16]$. ($f\circ g$ is the function $f(g(x))$.) *(5 points)*

   c) Find all zeros of the function $g\circ f$. *(4 points)*

3. From a regular tetrahedron, we cut off a corner with the plane through the points of the three edges from that vertex that are one third of the edge length away from the vertex. We did this at every vertex, and we got the polyhedron in the figure, with all edges 10 cm long.



   a) Find the surface area and the volume of the polyhedron. *(9 points)*

   b) What angle does a hexagonal face form with a neighboring triangular face? *(5 points)*

4. Decide whether the following statements A), B), and C) are true or false. Justify your answers.

   A) The segments connecting the centroid of a triangle to its vertices divide the triangle into three smaller triangles of equal area. *(6 points)*

   B) There is no quadratic equation with rational coefficients that has one rational root and one irrational root. *(4 points)*

   C) If we choose 30 days of a given year, it is not certain that there are at least five among them that fall on the same day of the week. *(4 points)*

Part II

5. The first term of the sequence a_n is $a_1 = 1$, and from the second term on, every term can be computed with the recursive formula $a_{n+1} = a_n + 2n$.

   a) Prove that $a_n = n^2 - n + 1$. *(3 points)*

   b) Find the median and the lower and upper quartiles of the first 11 terms of the sequence. *(2 points)*

   c) Prove (by induction or another method) that the sum of the first n terms of the sequence is $S_n = \frac{n(n^2+2)}{3}$. *(6 points)*

   We choose a positive integer $n < 1000$, and compute the average of the first n terms of the sequence.

   d) What is the probability that the average is an integer, if every positive integer $n < 1000$ is chosen with the same probability? *(5 points)*

6. The set H consists of the simple graphs whose five vertices are the vertices A, B, C, D, E of a given convex pentagon.

   a) How many elements does H have? *(3 points)*

   b) Draw all graphs in H with three edges that are pairwise not isomorphic. *(5 points)*

   c) How many graphs in H are connected and contain a cycle? *(8 points)*

7. The points with both coordinates integers on the circle of radius 5 centered at the origin determine an inscribed polygon.

   a) Find the angles of the polygon. *(8 points)*

   b) Find the second coordinates of the points of the y-axis from which the segment determined by the points $A\,(4, 5)$ and $B\,(8, -3)$ is seen at an angle of $45°$. *(8 points)*

8. a) Prove that the graph of the function $f\,(x) = \frac{1}{4}x^4 - \frac{1}{2}x^2 + \frac{1}{2}$ $(x \in \mathbb{R})$ has no common point with the x-axis. *(2 points)*

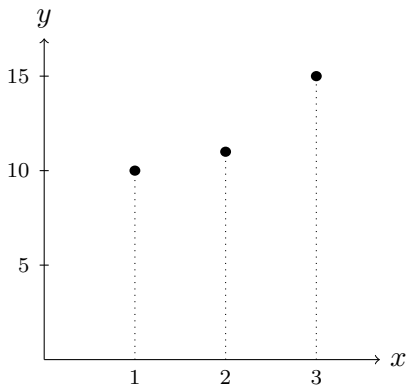
   b) Find the global and local extreme points and extreme values of f. *(6 points)*

   c) Find the area of the closed bounded region enclosed by the graph of f, the tangent line to the graph at the point with first coordinate $x_0 = 2$, and the two coordinate axes. *(8 points)*

9. In a survey about bottle return habits, a retail company asked 500 customers who had used a bottle return machine. At the machine, the refund can be requested as A) a store voucher, B) a bank transfer, or C) a charity donation. 398 customers chose A), 100 chose B), and 2 chose C).

   a) We choose 20 of the 500 customers at random. What is the probability that at most 16 of them chose option A)? Round your answer to four decimal places. *(5 points)*

   At one return point, 10 000 items were returned in the first week, 11 000 in the second week, and 15 000 in the third week. The figure shows these data; the unit on the vertical axis is a thousand items.



   b) Give a linear function whose graph passes through the point $M\,(2, 12)$, and whose values at $x_1 = 1$, $x_2 = 2$, $x_3 = 3$ have the smallest possible sum of squared differences from the values $y_1 = 10$, $y_2 = 11$, $y_3 = 15$. *(9 points)*

The returned items were sorted into three groups by material: a (plastic), b (metal), c (glass). Asked what kinds of bottles they usually return, 380 of the 500 customers above answered a, b, c; 50 answered a, b; 24 answered b, c; and 10 answered a, c. The customers who returned only one kind of item were divided among the types a, b, c in the ratio $3 : 2 : 1$.

c) How many customers returned bottles of type a? *(2 points)*

 Downloaded from martahidegkuti.com