Advanced Level Practice Set - 2025 (September 2025)

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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.
The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.
This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. a) Solve the following equation over the set of real numbers.

$$\cos\left(\frac{\pi}{6}\cos x\right) = \frac{\sqrt{3}}{2}$$

(5 points)

b) Find the positive prime numbers p and r in the equation $px + ry = 28$, if the pair of real numbers x, y is the solution of the system of equations $\sqrt{3x + y + 5} = 4x - 20$, $x^2 + y = 29$. *(7 points)*

2. The following table lists the names, clubs, and numbers of goals of 11 of the top scorers of the 2024–2025 season of the men's handball Champions League, counting the goals scored in the season until November 17, 2024.

name	club	goals
Kamil Syprzak	Paris Saint-Germain	112
Dika Mem	Barcelona	106
Nedim Remili	Veszprém	99
Emil Madsen	GOG	98
Ómar Ingi Magnusson	Magdeburg	98
Mats Hoxer	Aalborg	96
Aaron Mensing	GOG	93
Niclas Ekberg	THW Kiel	92
Mario Sostaric	OTP-Bank Pick Szeged	88
Mitja Janc	Celje	87
Elohim Prandi	Paris Saint-Germain	87

a) Compute the range, the mean, and the standard deviation of the sample (the numbers of goals) in the table. *(3 points)*

b) Find the mode (or modes), the median, and the lower and upper quartiles of the numbers of goals, and draw the box plot. *(4 points)*

We add to the data the result of Ludovic Fabregas, who plays for Veszprém. His number of goals is the two-digit prime number p, which is the smallest element of the new data set of 12 numbers. If the lower quartile of the new data set is Q_1' and its upper quartile is Q_3', then we also know that $6.5\,(Q_3' - Q_1') < p < 8.5\,(Q_3' - Q_1')$.

c) How many goals could Ludovic Fabregas have scored in the Champions League until the given date? *(5 points)*

3. a) Which integers do not satisfy the following inequality?

$$\log_2 \frac{(12x+4)(4x-3)}{15x-4} - \log_{0.5} \frac{15x-4}{4x-3} \leq \log_2 (30x-50)$$

(5 points)

The sides of a triangle are $4n-3$, $15n-4$, and $12n+4$, where n is a positive integer.

b) For which values of n is the triangle right, acute, or obtuse? *(8 points)*

4. A box contains 10 red balls, 13 green balls, and some blue balls. We pick a ball from the box at random.

a) What is the probability that the ball is not red, if we know that the number of blue balls is to the number of non-blue balls as the number of green balls is to the number of non-green balls? *(3 points)*

50 students reached the final round of a math competition. The jury awards 1 gold medal, 2 silver medals, and 3 bronze medals; every student can receive at most one medal.

b) In how many ways can the 6 medals be awarded? *(3 points)*

From the 44 students who did not receive a medal, the jury selects 20 whose good results it rates as equal, and a lottery decides which 6 of them receive a special prize. The names of the 20 students are written on slips of paper, and the slips are put into a box. 6 slips are drawn at random, without replacement.

c) Of the 20 students in the lottery, 8 came from school A, 7 from school B, and 5 from school C. What is the probability that at least 2 of the 6 names drawn are students of school A? *(8 points)*

Part II

5. a) If we subtract p times a positive three-digit number from the square of the number (where p is a positive prime), we get exactly $20p^2$. Find the smallest and the largest such three-digit number. *(7 points)*

G_1 and G_2 are trees with no common vertex. G_1 has $7q$ vertices and G_2 has $3r$ vertices, where q and r are positive primes. Connecting a vertex of G_1 to a vertex of G_2 by an edge, we get a tree G_3 with $q \cdot r$ edges.

b) Find the number of vertices and edges of G_1 and of G_2, and the sum of the degrees in each of the two graphs. *(9 points)*

6. We mark 28 points on a line in the plane. We also mark more than 7 points in the plane that are not on this line, so that no three of all the points are on a line, except for the 28 points on the line.

a) How many points did we mark outside the line, if the number of lines determined by all the marked points is 403? *(7 points)*

We mark n different points on a circle, where $n \geq 4$. Let A, B, and C denote the number of lines, triangles, and quadrilaterals determined by the n points.

b) Is it possible that $A + B = C$? (Justify your answer.) *(9 points)*

7. In the coordinate plane we mark the points $A(3,-4)$, $B(8,1)$, $C(6,5)$, $D(-1,-2)$, and $P(-2,1)$.

a) Prove that the five points are on a circle. *(4 points)*

b) Show that the quadrilateral $ABCD$ is a trapezoid, and compute its area. *(4 points)*

Consider the function $f : \mathbb{R} \to \mathbb{R}$, $f(x) = ax^2 + bx + c$ whose graph passes through the points A, B, and C, and the function $g : \mathbb{R} \to \mathbb{R}$, $g(x) = dx^2 + ex + f$ whose graph passes through the points A, C, and P.

c) Find the area of the region enclosed by the two graphs. *(8 points)*

8. a) Find the natural numbers n for which the inequality $n! > 3^n$ holds, and also those for which it does not hold. *(5 points)*

   Let $n > 1$ be a positive integer, and define the sequences a_n and b_n as follows:

$$a_n = \binom{n}{2} + \binom{n}{3}, \qquad b_n = \binom{2n}{3}$$

   b) Prove that the sequence $c_n = \frac{a_n}{b_n}$ is strictly decreasing. *(7 points)*

   The first term of the geometric sequence d_n is $d_1 = 24$ and its common ratio is $q_d = \frac{6}{22}$; the first term of the geometric sequence e_n is $e_1 = 21$ and its common ratio is $q_e = \frac{15}{22}$. Let S_d and S_e denote the infinite sums $d_1 + d_2 + d_3 + \cdots$ and $e_1 + e_2 + e_3 + \cdots$.

   c) Find the exact value of $S_d + S_e$. *(4 points)*

9. The legs of the right triangle ABC are $BC = a$ and $CA = b$, its hypotenuse is $AB = c$, and $\angle BAC = 15°$. We rotate the right triangle about the line of the leg BC, and also about the line of the leg CA.

   a) Find the exact value of the ratio of the volumes of the two solids we get. *(6 points)*

   In the acute triangle ABC, $BC = a$, $CA = b$, $AB = c$, and $a < b < c$. We rotate the triangle about the line of the side a, then about the line of the side b, and finally about the line of the side c. The volumes of the three solids, in this order, are three consecutive terms of an arithmetic sequence.

   b) Prove that the length of the side b is the harmonic mean of the lengths of the sides a and c. *(10 points)*

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