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Solutions: KoMaL website

From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.
The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.
This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. a) Solve the following system of equations over the set of pairs of real numbers.

$$\begin{aligned} \frac{5}{x+y}+\frac{4}{x-2y} &= \frac{3}{4} \\ \frac{15}{x+y}-\frac{8}{x-2y} &= 1 \end{aligned}$$

(9 points)

b) A two-digit number leaves remainder 2 when divided by 5, remainder 3 when divided by 6, and remainder 6 when divided by 9. Find the two-digit number. *(5 points)*

Solution: a) Because of the denominators, $x+y \neq 0$ and $x-2y \neq 0$. With the new unknowns $a=\frac{1}{x+y}$ and $b=\frac{1}{x-2y}$ the system becomes

$$\begin{aligned} 5a+4b &= \frac{3}{4} \\ 15a-8b &= 1 \end{aligned}$$

Multiplying the first equation by 2 and adding it to the second, $25a=\frac{3}{2}+1=\frac{5}{2}$, so $a=\frac{1}{10}$. Substituting into the first equation, $\frac{5}{10}+4b=\frac{3}{4}$, so $4b=\frac{1}{4}$ and $b=\frac{1}{16}$. So

$$x+y=10, \qquad x-2y=16$$

Subtracting the second equation from the first, $3y=-6$, so $y=-2$, and then $x=12$. Check:

$$\frac{5}{10}+\frac{4}{16}=\frac{1}{2}+\frac{1}{4}=\frac{3}{4}, \qquad \frac{15}{10}-\frac{8}{16}=\frac{3}{2}-\frac{1}{2}=1$$

So the solution of the system is $x=12$, $y=-2$.

b) The number 3 more than the number in question is divisible by 5, 6, and 9, so by their least common multiple 90. The only such two-digit number is 90, so the number in question can only be 87, which indeed satisfies the conditions.

2. Ferenc Szatmári and his family traveled by car from Budapest to Kisvárda on a hot summer day. They left at 7:50. In 25 minutes, after 12 km, they reached the M3 highway. At 10:55 they left the highway for road 403, and after another 40 km, having driven 284 km in all, they arrived in Kisvárda at 11:35.

a) What was the average speed of the car on the highway? *(3 points)*

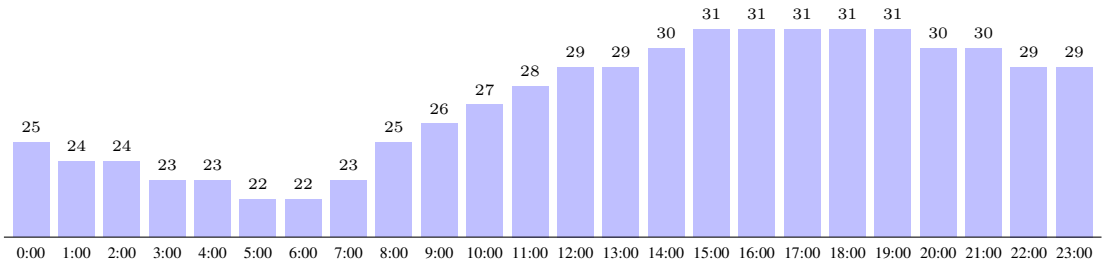
On the M3, at five places, there were speed limits because of road repairs, on 20 km in all. On these sections the speed limit was 80 km/h, but the drivers could only drive at 50 km/h.

b) Ferenc thought that if he could have driven at 120 km/h here, they would have arrived in Kisvárda at least half an hour earlier. Is he right? *(3 points)*

The three children of the family, Péter, Anna, and Lilla, traveled on the back seat of the car, where three people may sit. Péter asked not to sit in the middle; he does not like to travel between the two girls.

c) In how many ways could they sit on the three back seats, if they respected Péter's request? *(3 points)*

The hourly weather forecast for that day in a weather app (temperatures in °C):



d) Find the mode and the median of the hourly temperature values of this day. What is the average temperature based on the hourly data? *(4 points)*

Solution: a) The distance on the M3 is $284 - 12 - 40 = 232$ km. They reached the highway at 8:15, and from 8:15 to 10:55, 2 hours 40 minutes, that is, $2\frac{2}{3}$ hours passed. The average speed was

$$\frac{232 \text{ km}}{2\frac{2}{3} \text{ h}} = 87 \text{ km/h}$$

b) At 50 km/h the 20 km takes $\frac{20}{50} = 0.4$ hours $= 24$ minutes, so they could not have saved 30 minutes in any case. Ferenc is wrong. (At 120 km/h the 20 km would take 10 minutes, a saving of only 14 minutes.)

c) Péter can sit on the left or the right seat: 2 possibilities. Then Anna can take either of the two remaining seats, and Lilla sits on the last one. So there are $2 \cdot 2 = 4$ possibilities.

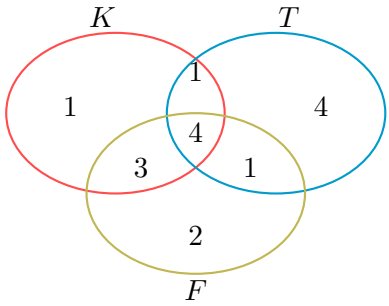
d) The frequencies of the temperature values are

temperature (°C)	22	23	24	25	26	27	28	29	30	31
frequency	2	3	2	2	1	1	1	4	3	5

So the mode is 31°C. There are 24 data values; in increasing order the 12th one is 28°C and the 13th one is 29°C, so the median is $\frac{28+29}{2} = 28.5$°C. The average temperature is

$$\frac{2 \cdot 22 + 3 \cdot 23 + 2 \cdot 24 + 2 \cdot 25 + 26 + 27 + 28 + 4 \cdot 29 + 3 \cdot 30 + 5 \cdot 31}{24} = \frac{653}{24} \approx 27.2°\text{C}$$

3. The 30 students of class 11a took part in many kinds of summer programs. Among others, they traveled abroad (K), were at a camp at Lake Balaton (T), and went to a pop music festival (F). The Venn diagram shows how many students took part in which programs.



a) How many students went neither abroad, nor to the camp, nor to the festival? *(2 points)*

b) We choose a student of the class at random. What is the probability that the student was abroad or at the camp, but not at the festival? *(3 points)*

Reading the required and recommended books is not always easy. In a school year, the reading list can contain as many as 15 literary works.

c) Soma plans to read a required book of 405 pages in exactly 15 days as follows. From the second day on, he reads one page more each day than on the day before. How many pages does he plan to read on the first day, and how many on the fifteenth day? *(5 points)*

7 students came to a meeting of a math club. Their math teacher usually starts the club with a conversation.

d) The teacher asked them how many of the students present they had called on their cell phones the previous week. They said these numbers: Péter 4, Sári 1, Balázs 0, Sanyi 2, Bori 2, Ernő 4, Miki 2. The teacher said that somebody remembered wrong. What did the teacher base this opinion on? *(2 points)*

Solution: a) The sum of the numbers in the Venn diagram, $1+1+4+3+4+1+2=16$, is the number of students who took part in at least one of the three programs. The class has 30 students, so $30-16=14$ students went neither abroad, nor to the camp, nor to the festival.

b) We need the number of elements of the set $(K \cup T) \setminus F$. Adding the numbers in the corresponding regions of the diagram, $1+1+4=6$. So there are 6 favorable cases out of 30, and the probability is $\frac{6}{30}=\frac{1}{5}=0.2$.

c) Soma reads x pages on the first day and $x+14$ pages on the last day, 405 pages in the 15 days:

$$\frac{(x+x+14)\cdot 15}{2}=405, \qquad 2x+14=54, \qquad x=20$$

So Soma plans to read 20 pages on the first day and 34 pages on the fifteenth day.

d) Every phone call is mentioned by both people who took part in it, so the sum of the numbers should be even. The sum of the numbers is $4+1+0+2+2+4+2=15$, which is odd, so indeed somebody remembered wrong.

Remark: We can draw this as a graph: the vertices are the students, and an edge means that the two students talked on the phone. In a graph the sum of the degrees is even, or in other words, the number of vertices with odd degree is even. The graph described by the students does not have this property.

4. Prove the following statements.

a) $\binom{2024}{10} + 2\binom{2024}{11} + \binom{2024}{12} = \binom{2026}{12}$ *(4 points)*

b) If n is a positive integer, then $(n-1)! + n! + (n+1)! = (n+1)! \cdot \frac{n+1}{n}$. *(4 points)*

c) $\frac{3^{150} - 2 \cdot 3^{149} + 3^{148}}{3^{147} + 3^{146}} = 9$ *(4 points)*

Solution: a) By the property of Pascal's triangle, $\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$, so

$$\binom{2024}{10} + 2\binom{2024}{11} + \binom{2024}{12} = \binom{2024}{10} + \binom{2024}{11} + \binom{2024}{11} + \binom{2024}{12} = \binom{2025}{11} + \binom{2025}{12} = \binom{2026}{12}$$

b) We express $n!$ and $(n+1)!$ with $(n-1)!$:

$$\begin{aligned}(n-1)! + n! + (n+1)! &= (n-1)! + n\,(n-1)! + (n+1)\,n\,(n-1)! = (1 + n + (n+1)\,n)\,(n-1)! \\ &= (n+1)^2\,(n-1)! = \frac{(n+1)\,(n+1)\,n\,(n-1)!}{n} = (n+1)! \cdot \frac{n+1}{n}\end{aligned}$$

c) Using the rules of exponents in the numerator and the denominator,

$$\frac{3^{150} - 2 \cdot 3^{149} + 3^{148}}{3^{147} + 3^{146}} = \frac{9 \cdot 3^{148} - 6 \cdot 3^{148} + 3^{148}}{3 \cdot 3^{146} + 3^{146}} = \frac{4 \cdot 3^{148}}{4 \cdot 3^{146}} = 3^2 = 9$$

Part II

5. a) Solve the equation $\left(\sin x - \frac{\sqrt{3}}{2}\right)\left(\cos x + \frac{\sqrt{2}}{2}\right) = 0$ over the set of real numbers. *(7 points)*

b) Find the range of the function $f : \mathbb{R} \to \mathbb{R}$, $x \mapsto \frac{1}{\sin x + 2}$. *(4 points)*

A store sells four kinds of pocket calculators: the PÖPEC brand for 9500 or 6100 forints, and the SPICC brand for 5800 or 7100 forints. According to the store's statistics, 60% of the customers buy the PÖPEC brand and 40% choose SPICC. 40% of the customers who buy PÖPEC choose the more expensive model, and in the case of SPICC, 65% choose the more expensive one.

c) What percent of the customers buy a calculator cheaper than 7000 forints? *(5 points)*

Solution: a) A product is 0 exactly when one of its factors is 0.

If $\sin x - \frac{\sqrt{3}}{2} = 0$, then $\sin x = \frac{\sqrt{3}}{2}$, so $x = \frac{\pi}{3} + 2k\pi$ or $x = \frac{2\pi}{3} + 2l\pi$, where k and l are integers.

If $\cos x + \frac{\sqrt{2}}{2} = 0$, then $\cos x = -\frac{\sqrt{2}}{2}$, so $x = \frac{3\pi}{4} + 2m\pi$ or $x = \frac{5\pi}{4} + 2n\pi$, where m and n are integers.

The period of $\sin x$ and $\cos x$ is 2π, so it is enough to check the solutions for $k = l = m = n = 0$; they indeed satisfy the equation.

b) The range of $\sin x$ is the interval $[-1, 1]$, so the range of $\sin x + 2$ is $[1, 3]$. So the range of $\frac{1}{\sin x + 2}$ is the interval $\left[\frac{1}{3}, 1\right]$, since every number in $\left[\frac{1}{3}, 1\right]$ is the reciprocal of a number in $[1, 3]$.

c) The calculators cheaper than 7000 forints are the 5800 forint SPICC and the 6100 forint PÖPEC. 40% of the customers choose SPICC, and $100\% - 65\% = 35\%$ of them buy the cheaper model: $0.4 \cdot 0.35 = 0.14$ of the customers. 60% of the customers choose PÖPEC, and $100\% - 40\% = 60\%$ of them buy the cheaper model: $0.6 \cdot 0.6 = 0.36$ of the customers. So $0.14 + 0.36 = 0.5$, that is, 50% of the customers buy a calculator cheaper than 7000 forints.

6. Consider the function $f : [0, 4] \to \mathbb{R}$, $x \mapsto \frac{1}{4}(x-4)^2$.

   a) Find the coordinates of the common points of the graph of the function and the coordinate axes. *(4 points)*

   b) Find the area of the region enclosed by the graph of the function and the coordinate axes. *(6 points)*

   c) Find the inverse of the function f. *(6 points)*

   Solution: a) The points on the x-axis have second coordinate 0. If $\frac{1}{4}(x-4)^2 = 0$, then $x = 4$: the graph touches the x-axis at the point $A(4, 0)$. The points on the y-axis have first coordinate 0, and $f(0) = \frac{1}{4}(0-4)^2 = 4$: the graph meets the y-axis at the point $B(0, 4)$.

   b) We compute the area with a definite integral on the interval of part a):

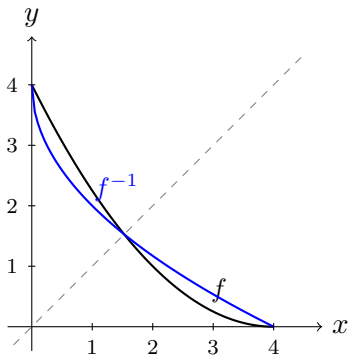
$$T = \int_0^4 \frac{1}{4}(x-4)^2\,dx = \left[\frac{1}{12}(x-4)^3\right]_0^4 = 0 - \frac{(-4)^3}{12} = \frac{16}{3}$$

   square units.

   Note: KoMaL's solution has a misprint in the middle step of this computation; the result is correct.

   c) $f(0) = 4$ and $f(4) = 0$, and f is continuous and decreasing, so its range is the interval $[0, 4]$; this is the domain of the inverse function. From $y = \frac{1}{4}(x-4)^2$ we get $4y = (x-4)^2$ with $y \geq 0$, so $2\sqrt{y} = |x-4|$, and since $0 \leq x \leq 4$, $|x-4| = 4-x$. So $2\sqrt{y} = 4-x$, that is, $x = 4 - 2\sqrt{y}$. Writing x for the variable of the inverse function too:

$$f^{-1} : [0, 4] \to \mathbb{R}, \qquad x \mapsto 4 - 2\sqrt{x}$$



7. The equation of the line e is $4x + 3y = 120$ and the equation of the line f is $3x - 4y = -60$. In the quadrilateral $ABCD$, the vertex A is the point $(0, 0)$, the vertex B is on the x-axis, and the vertex D is on the y-axis. The side BC is on the line e and the side CD is on the line f.

   a) Find the coordinates of the vertices B, C, D of the quadrilateral. *(4 points)*

   b) Prove that the quadrilateral is cyclic. *(5 points)*

   c) Write the equation of the circumscribed circle of the quadrilateral. *(4 points)*

   We found a practice problem set with 25 problems on the internet. 8 of the problems are about sequences, 6 about functions, 4 about combinatorics, and 7 about coordinate geometry.

   d) We choose five of these problems at random, at the same time, so that every problem is chosen with the same chance. What is the probability that there are 3 problems about sequences and 2 problems about coordinate geometry among the chosen ones? *(3 points)*

   Solution: a) B is on the x-axis, so its second coordinate is 0, and it is on the line e: $4x + 3 \cdot 0 = 120$, so $x = 30$ and $B(30, 0)$. D is on the y-axis and on the line f: $x = 0$ and $3 \cdot 0 - 4y = -60$ give $y = 15$, so $D(0, 15)$. C is the intersection of the lines e and f; we solve the system

$$4x + 3y = 120, \qquad 3x - 4y = -60$$

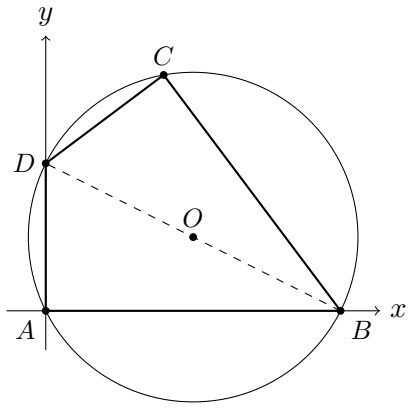
Multiplying the first equation by 4 and the second by 3, and adding them, $25x = 300$, so $x = 12$, and then $y = 24$: $C\,(12, 24)$.

b) The x- and y-axes are perpendicular, so the quadrilateral has a right angle at A. Also,

$$\overrightarrow{CB} = (18, -24) \quad \text{and} \quad \overrightarrow{DC} = (12, 9)\,, \qquad \overrightarrow{CB} \cdot \overrightarrow{DC} = 18 \cdot 12 + (-24) \cdot 9 = 0$$

so the two vectors are perpendicular, and the angle at C is also a right angle. So two opposite angles of the quadrilateral add up to 180°, and the quadrilateral is cyclic.

c) The triangles ABD and BCD are right triangles, so by the converse of Thales' theorem, A and C are on the circle with diameter BD. This is the circumscribed circle of the quadrilateral.



The center of the circle is the midpoint of BD, $O\left(\frac{30+0}{2}, \frac{0+15}{2}\right) = O\,(15, 7.5)$, and its radius is

$$r = OB = \sqrt{(30-15)^2 + (0-7.5)^2} = \sqrt{281.25}$$

So the equation of the circle is

$$(x-15)^2 + (y-7.5)^2 = 281.25$$

d) Five of the twenty-five problems can be chosen in $\binom{25}{5}$ ways. Three of the sequence problems can be chosen in $\binom{8}{3}$ ways, and two of the coordinate geometry problems in $\binom{7}{2}$ ways. So the probability is

$$P = \frac{\binom{8}{3}\binom{7}{2}}{\binom{25}{5}} = \frac{56 \cdot 21}{53\,130} = \frac{1176}{53\,130} \approx 0.022$$

8. A company growing fir trees made a plan for the three years from 2025 to 2027. By their estimate, they had $10\,000$ fir trees on their land in September 2025. Every year they want to plant 2000 new trees in mid-October, and cut down 10% of the trees in December.

a) How many trees do they plan to cut down in these three years in all? *(5 points)*

Let a_n be the planned number of trees at the end of the nth year, that is, at the end of the year $2024 + n$. We assume that they manage the trees by the same principles in the following years too.

b) Find the general term of the sequence a_n. *(6 points)*

At one market, three kinds of fir trees have been competing for the customers' favor before Christmas for years: the Nordmann fir, the Norway spruce, and the silver fir. 45% of the customers choose the Nordmann fir, 37% the Norway spruce, and 13% the silver fir. Other kinds of trees account for 5% of the sales.

c) If we ask six randomly chosen people who bought a tree what kind of tree they bought, what is the probability that three of them bought a Nordmann fir, two a Norway spruce, and one a silver fir? Give the probability to three decimal places. (We assume that everybody bought exactly one tree.) *(5 points)*

Solution: a) At the end of October 2025 there are 12 000 trees. In December they cut down 10% of them, 1200 trees, and 90% of them, 10 800 trees, remain. Continuing in the same way:

year	after planting	cut down	end of the year
2025	12 000	1200	10 800
2026	12 800	$12\,800 \cdot 0.1 = 1280$	$12\,800 \cdot 0.9 = 11\,520$
2027	13 520	$13\,520 \cdot 0.1 = 1352$	$13\,520 \cdot 0.9 = 12\,168$

So they plan to cut down $1200 + 1280 + 1352 = 3832$ trees.

Note: KoMaL's table writes $12\,000 \cdot 0.1$ *and* $12\,000 \cdot 0.9$ *in the row of* 2026 *instead of* $12\,800 \cdot 0.1$ *and* $12\,800 \cdot 0.9$*; the values are correct.*

b) $a_1 = 12\,000 \cdot 0.9$ and $a_2 = 12\,000 \cdot 0.9^2 + 2000 \cdot 0.9$. By the rule $a_n = (a_{n-1} + 2000) \cdot 0.9$,

$$a_n = 12\,000 \cdot 0.9^n + 2000 \cdot 0.9^{n-1} + 2000 \cdot 0.9^{n-2} + \cdots + 2000 \cdot 0.9$$

From the second term on, this is the sum of the first $n-1$ terms of a geometric sequence with first term $2000 \cdot 0.9$ and ratio 0.9:

$$a_n = 12\,000 \cdot 0.9^n + 2000 \cdot 0.9 \cdot \frac{0.9^{n-1} - 1}{0.9 - 1} = 12\,000 \cdot 0.9^n + 18\,000\left(1 - 0.9^{n-1}\right)$$

which can be simplified to

$$a_n = 18\,000 - 8000 \cdot 0.9^n$$

(Check: $a_1 = 10\,800$, $a_2 = 11\,520$, $a_3 = 12\,168$.)

c) The probability that the first three people bought a Nordmann fir, the next two a Norway spruce, and the sixth a silver fir is $0.45^3 \cdot 0.37^2 \cdot 0.13$. The three Nordmann buyers can be chosen from the six people in $\binom{6}{3}$ ways, and the two Norway spruce buyers from the remaining three in $\binom{3}{2}$ ways; the last person bought the silver fir. So the probability is

$$\binom{6}{3}\binom{3}{2} \cdot 0.45^3 \cdot 0.37^2 \cdot 0.13 = 60 \cdot 0.45^3 \cdot 0.37^2 \cdot 0.13 \approx 0.097$$

9. a) Solve the inequality $\lg\left(36 + 2^{\sqrt{3x}}\right) > 2$ over the set of real numbers. ($\lg$ denotes the base 10 logarithm.) *(7 points)*

b) State the converse of the following statement. Decide whether the statement and its converse are true.

If a real number is greater than 5, then the base 10 logarithm of the number is defined. *(3 points)*

Part II of an advanced-level school-leaving exam has five problems, numbered 5 to 9. The students have to solve only four of them.

c) In a math group of 17 students, everybody wrote the advanced-level exam. Péter and Anna are talking.

Péter: It is certain that at least four of us left out the same problem.

Anna: It is certain that there was a problem that exactly four of us left out.

Whose statement is true? Justify your answer. *(4 points)*

The table below shows how many students chose each problem and, of those who worked on it, how many solved it perfectly, for 16 points.

	problem 5	problem 6	problem 7	problem 8	problem 9
worked on it	14	14	16	13	11
16 points	4	6	9	3	5

d) What is the probability that if we choose one of the 17 exam papers at random, we see that the student solved problem 8 for 16 points? *(2 points)*

Solution: a) The square root is defined only for nonnegative numbers, so $\sqrt{3x}$ is defined if $x \geq 0$. Since $36 + 2^{\sqrt{3x}} \geq 36 + 2^0 > 0$, the logarithm is always defined when $x \geq 0$. So we look for the solutions with $x \geq 0$.

$$\lg\left(36 + 2^{\sqrt{3x}}\right) > 2 = \lg 100$$

The base 10 logarithm is strictly increasing, so $36 + 2^{\sqrt{3x}} > 100$, that is, $2^{\sqrt{3x}} > 64 = 2^6$. The function 2^x is strictly increasing, so $\sqrt{3x} > 6$. Both sides are nonnegative, so squaring is an equivalent transformation: $3x > 36$, that is, $x > 12$. This satisfies the condition $x \geq 0$, so the solution of the inequality is $x > 12$.

b) The converse of the statement: if the base 10 logarithm of a real number is defined, then the number is greater than 5. Any positive number not greater than 5 is a counterexample. So the original statement is true and its converse is false.

c) We prove Péter's statement indirectly. Every student left out exactly one problem. If every problem had been left out by at most three students, then there would be at most $5 \cdot 3 = 15$ students in the group. So some problem was left out by at least four students: Péter is right.

Anna is not right: for example, it is possible that everybody left out problem 9.

d) In 3 of the 17 papers, problem 8 has the maximal score. So the probability is $P = \frac{3}{17} \approx 0.176$.

Remark: $\frac{13}{17}$ of the students worked on problem 8, and $\frac{3}{13}$ of these got 16 points for it. So the probability that the chosen student got 16 points for problem 8 is $\frac{13}{17} \cdot \frac{3}{13} = \frac{3}{17} \approx 0.176$.

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