

Problem Set I - 1981 (December)

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For high school students preparing for universities and colleges, one of the greatest challenges is the entrance examination. KoMaL wants to help them prepare by publishing problem sets that are similar in difficulty to the written entrance examination. This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time.

1. Solve the equation $x^2 - x = a^2 - \frac{1}{4}$, where a is a real parameter.
2. The legs a and b and the hypotenuse c of a right triangle satisfy $(a - b)^2 = c^2 - b^2$. Find the altitude belonging to the hypotenuse.
3. The line $3x + 4y = 12$ intersects the coordinate axes at points A and B . The perpendicular bisector of segment AB splits triangle AOB into two parts. Find the areas of these parts. (O is the origin.)
4. Solve the given equation.

$$\log_{\sin x} (1 - \cos^2 x) = 2^{\sqrt{4-x^2}}$$

5. For what values of the parameter α does the equation $\sin^4 x + \cos^4 x = \sin^2 \alpha$ have a solution?
6. One of the acute angles of a right triangle is 30° and its hypotenuse is 4 units long. Find the area of the triangle whose vertices are the points where the inscribed circle is tangent to the sides.
7. Find all integers p and q that satisfy the equation $p^3 + 6p^2 + 5p = 3q^2 + 3q + 1$.
8. Find all pairs of numbers (x, y) that satisfy the given equation.

$$\sqrt{x + y - \sqrt{x + y - 1}} + \sqrt{x^2 - x + 1} = \frac{\sqrt{3}}{2}$$